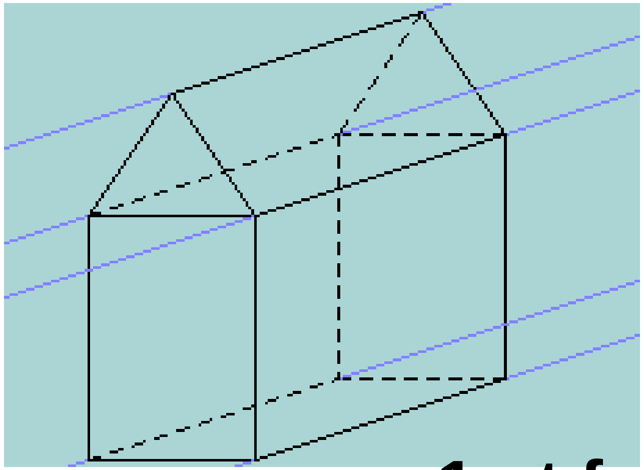
Journal club - Persival

Koenderink, J. J., & van Doorn, A. J. (2003).
Perspectives on colour space.
Colour Perception: Mind and the Physical World,
pp 1-56.

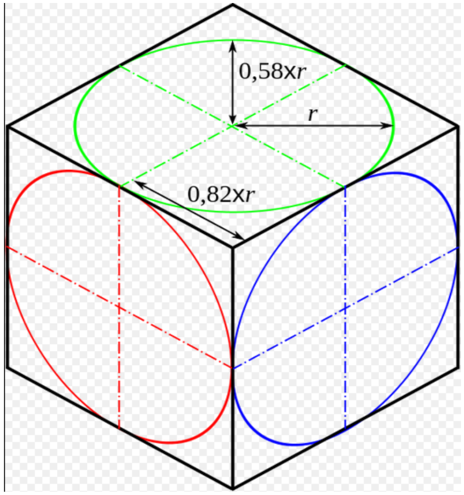
Objective

Attach **colour** vision a geometry like we do with space through **Perspectives**

Cavalière

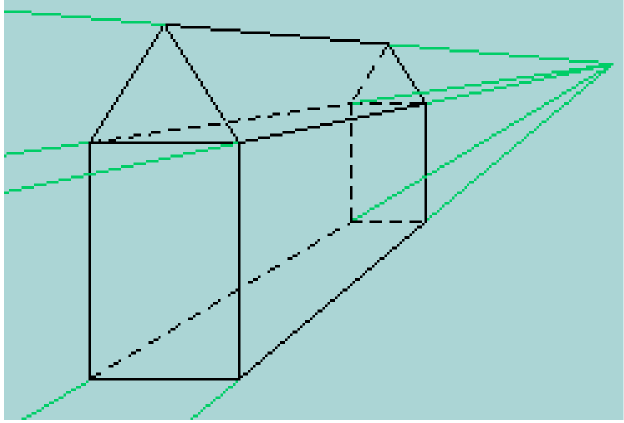


Isométrique

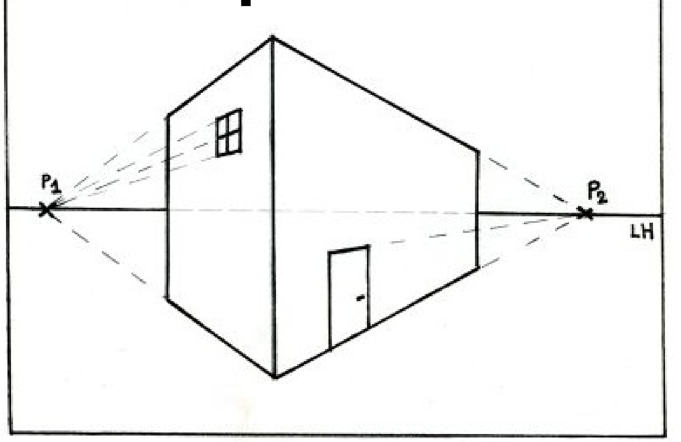


Art

1 pt fuite

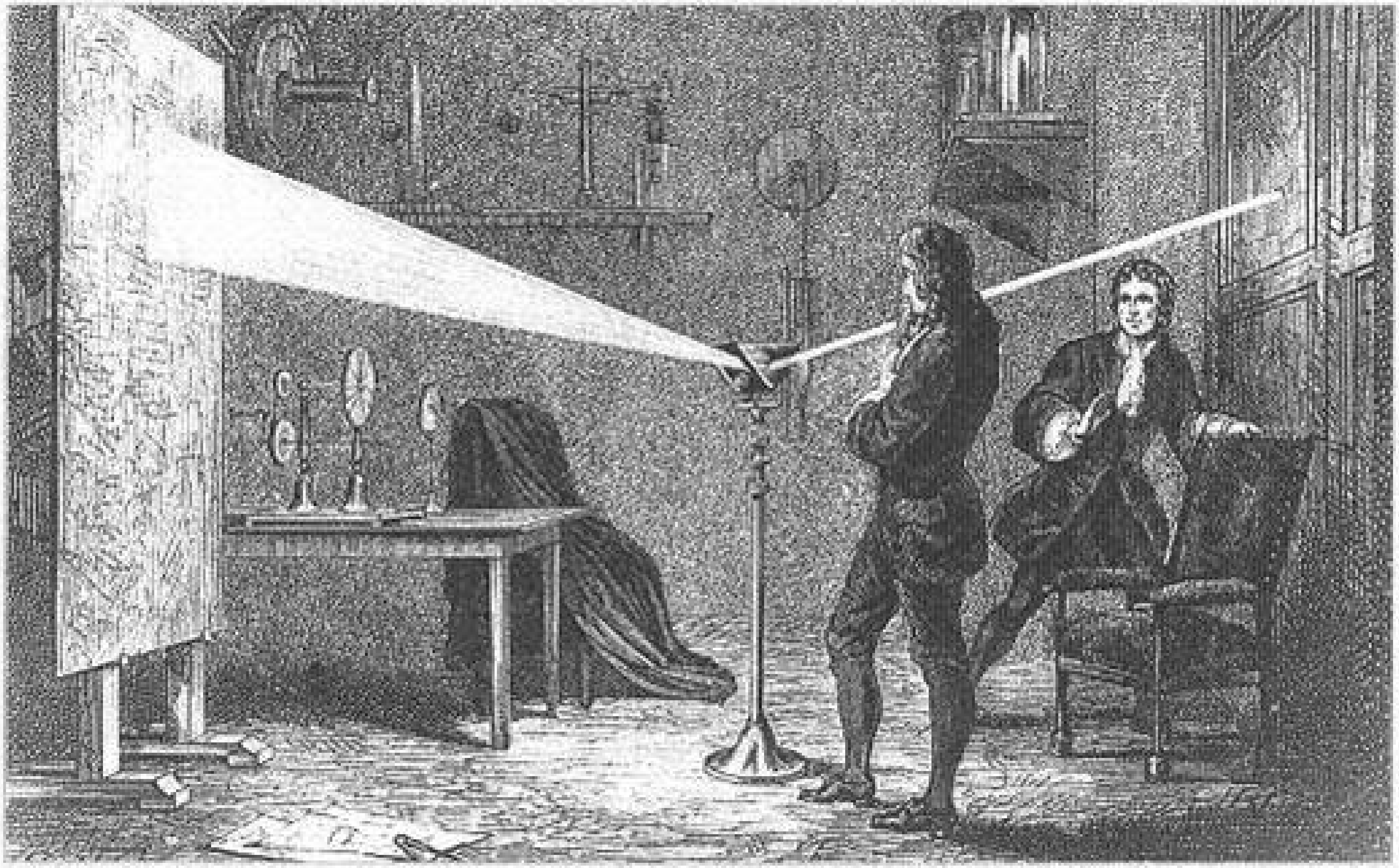


2 pt fuite



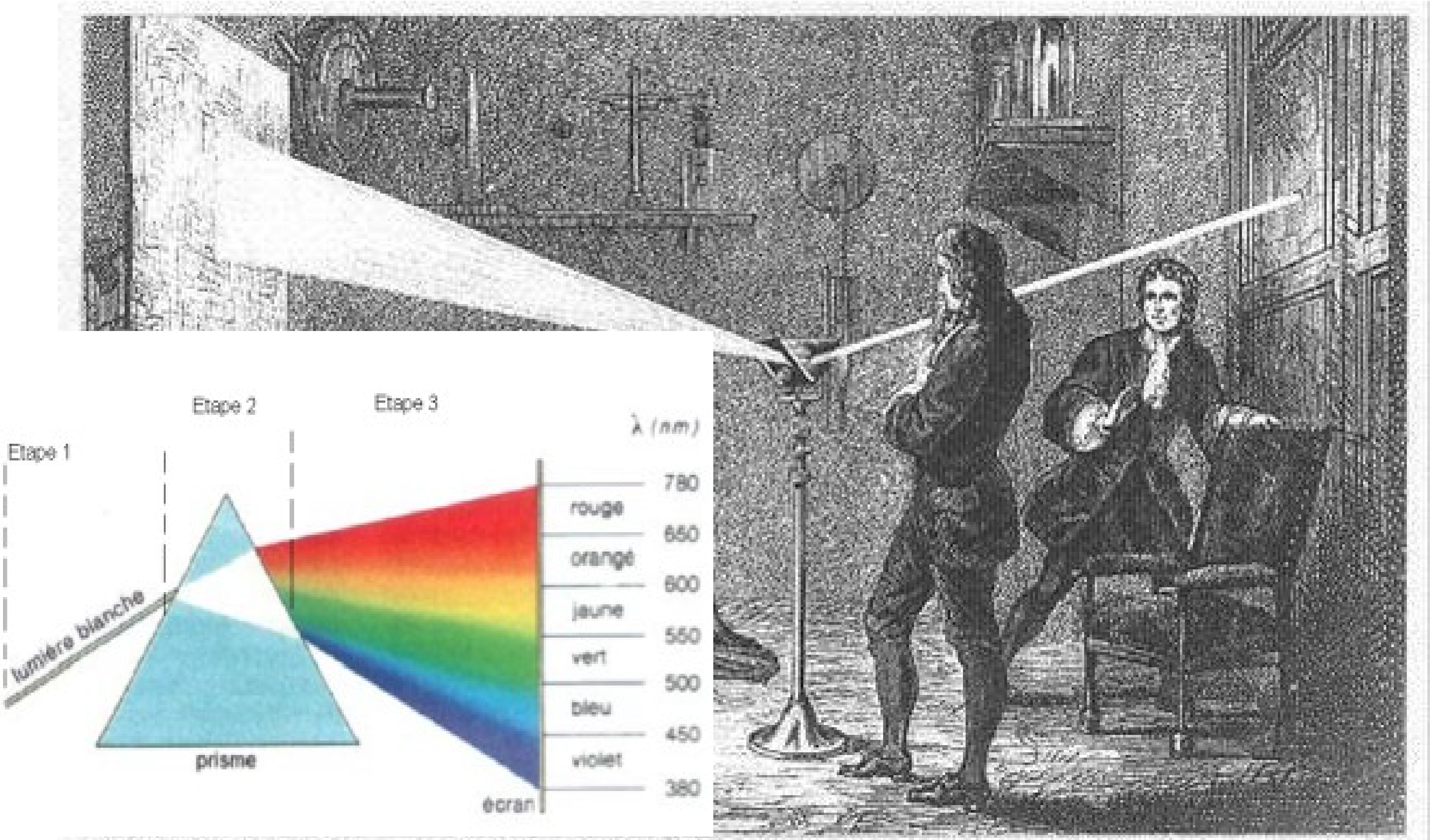
Newton

(1643 - 1727)



Newton en train de réaliser l'expérience des couleurs (1666). (Gravure du XIX^{e} siècle.)

Newton



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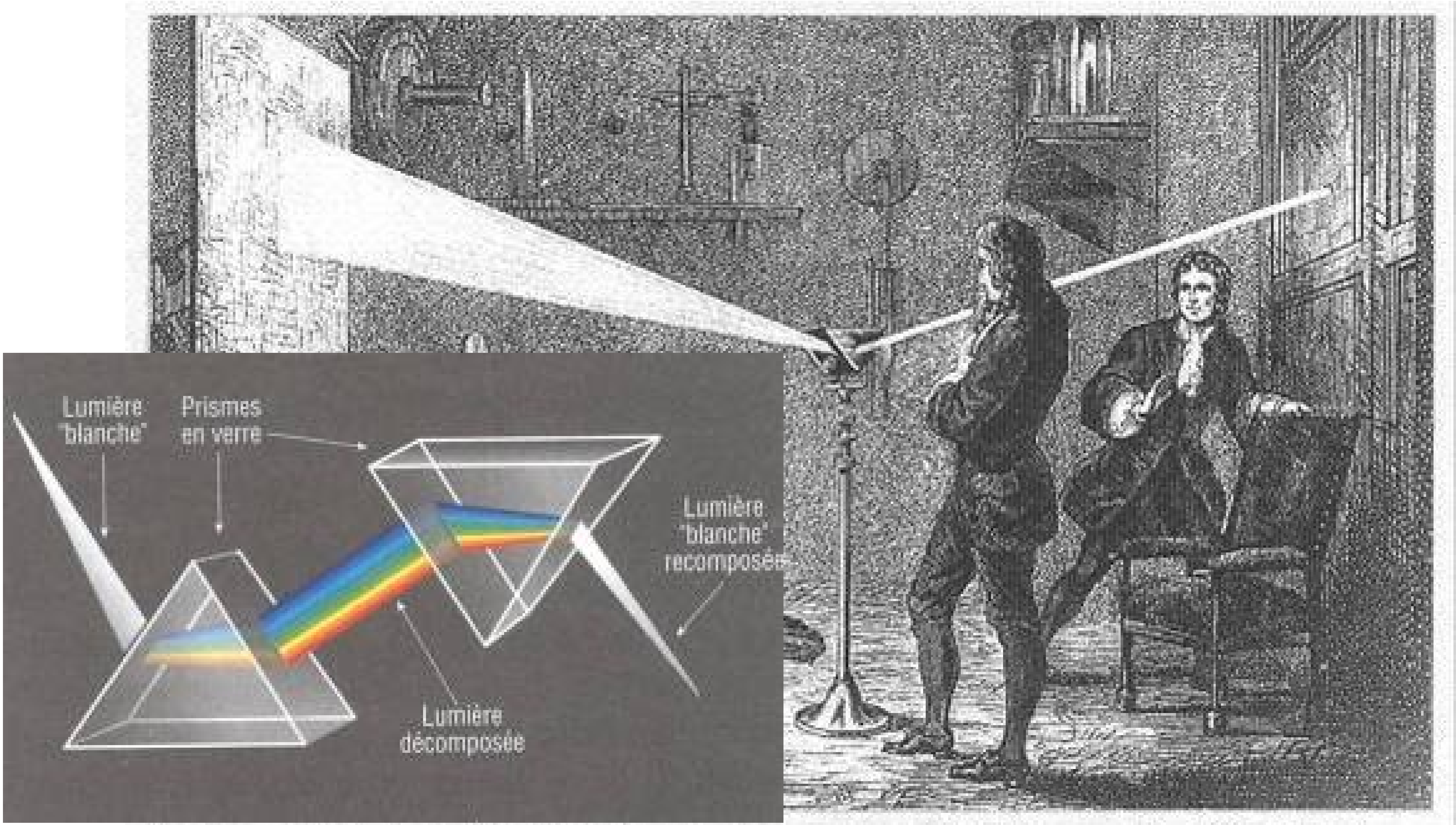
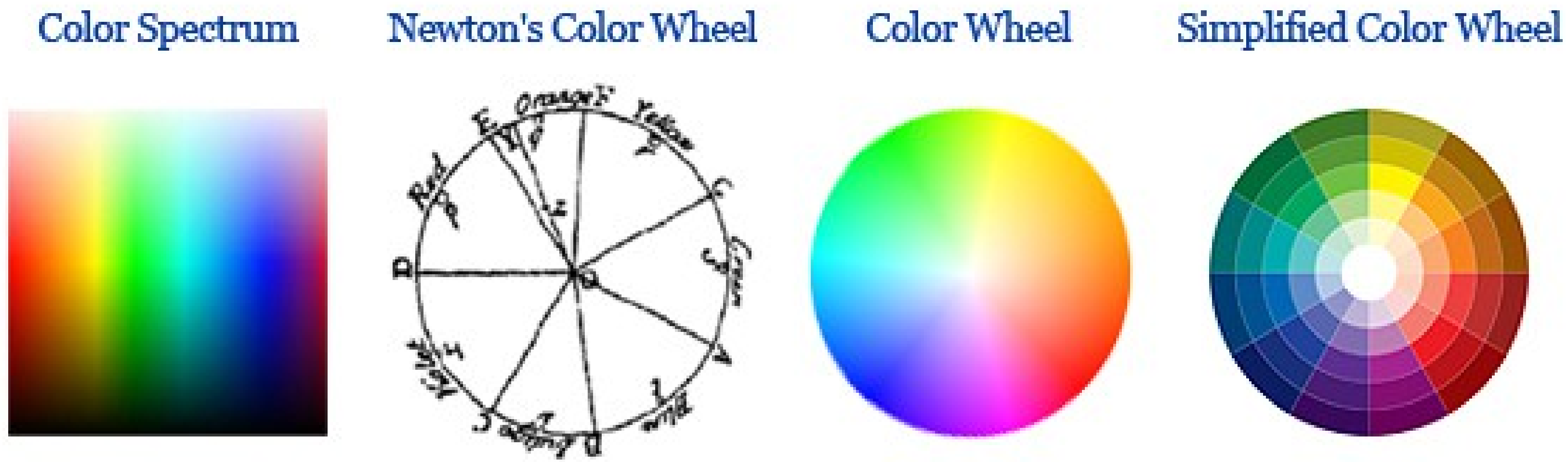
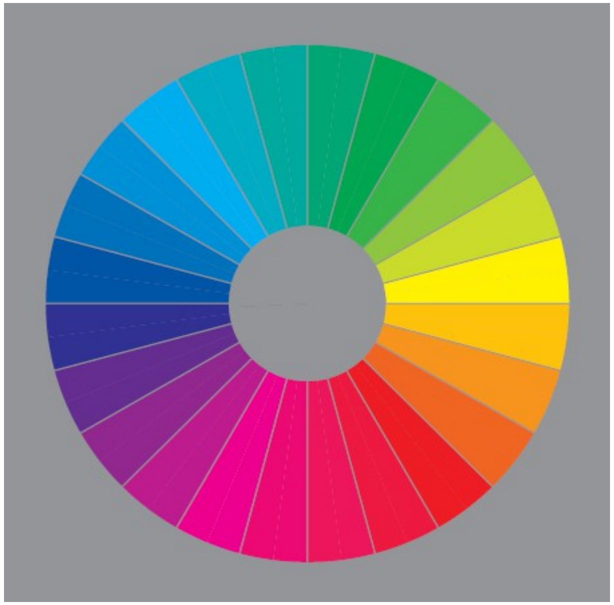
Newton



Newton en train de réaliser l'expérience des couleurs (1666). (Gravure du XIX^e siècle.)

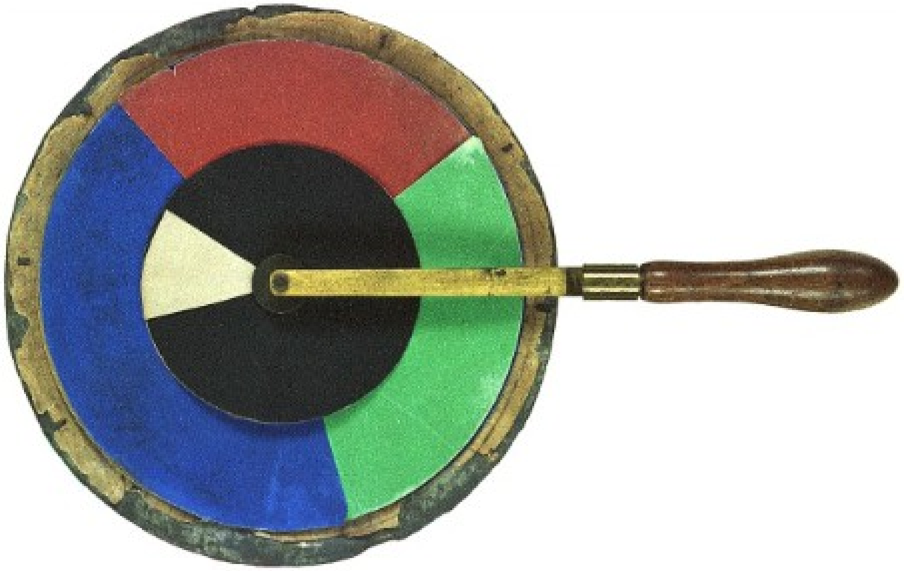
Newton

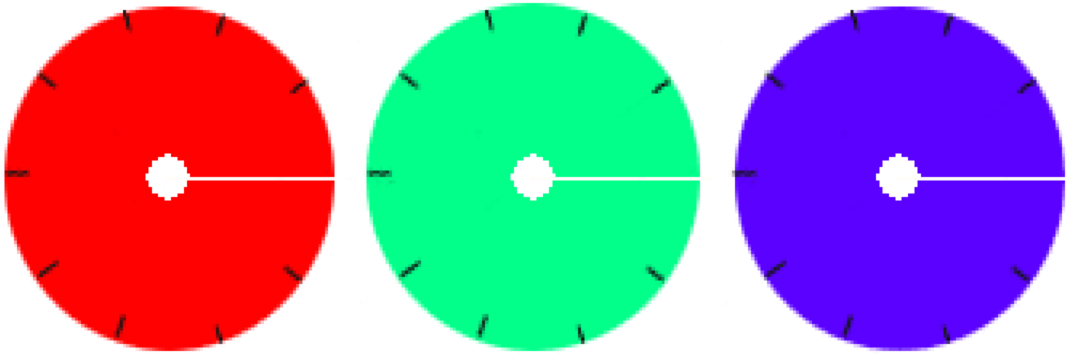

Color Spectrum
Newton's Color Wheel
Color Wheel
Simplified Color Wheel
Orange
Yellow
Green
Blue
Indigo
Violet
Red
D
E
F
G
A
B
C
O



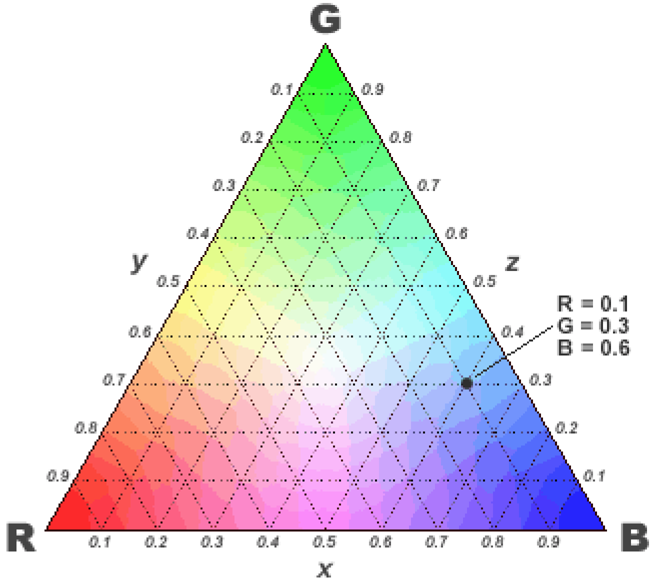
Maxwell

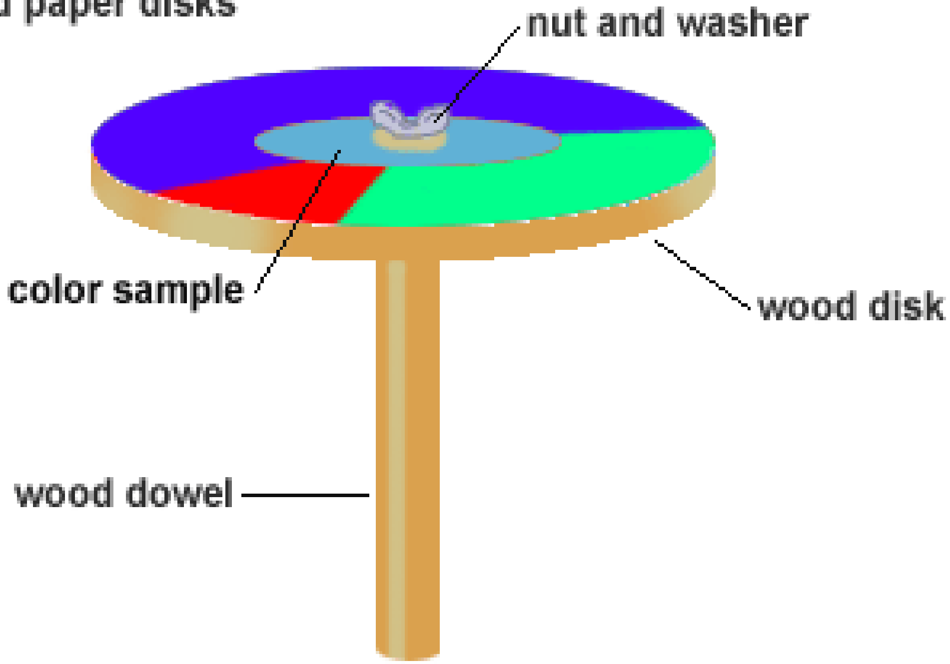
(1831 - 1879)



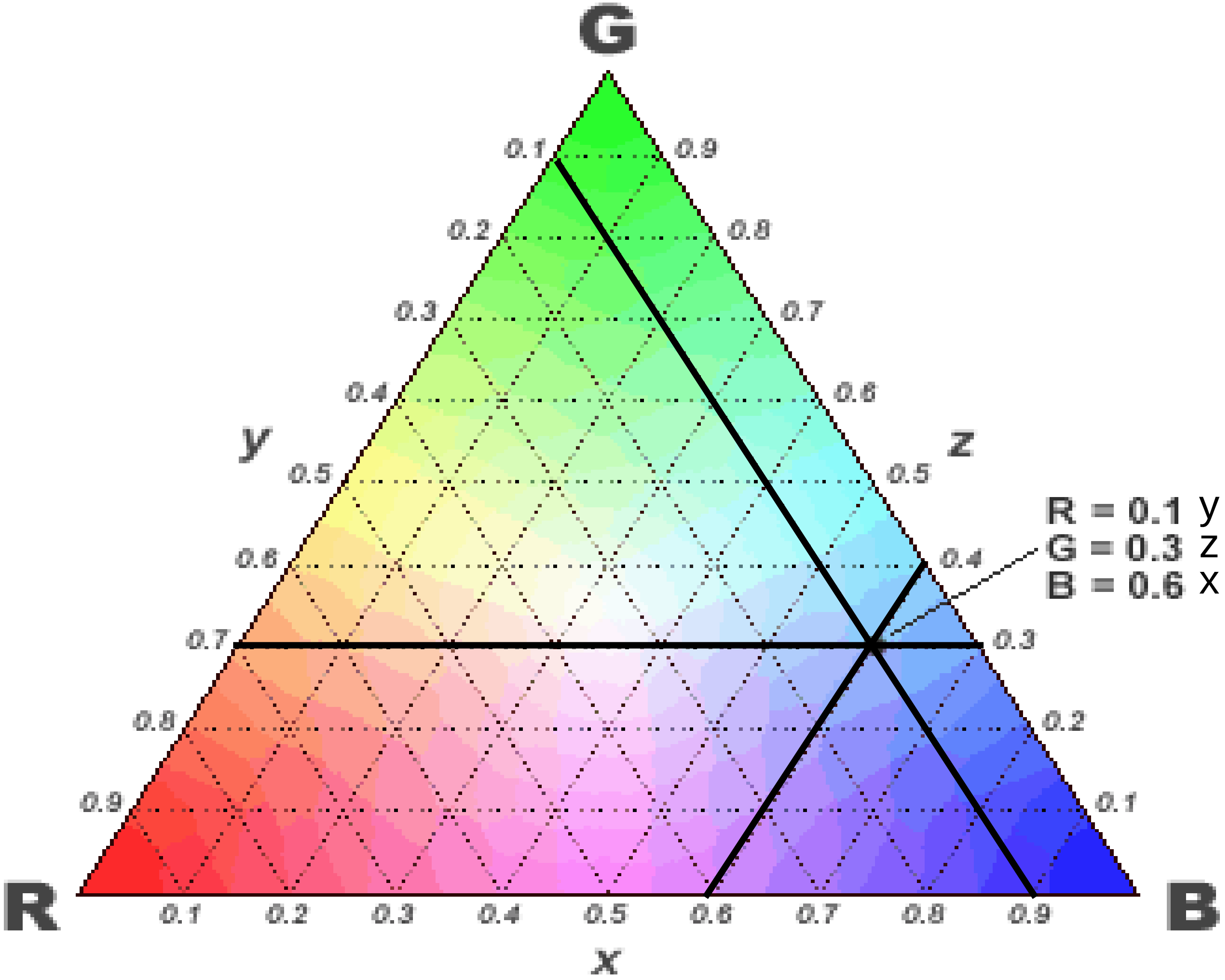


painted paper disks





Maxwell

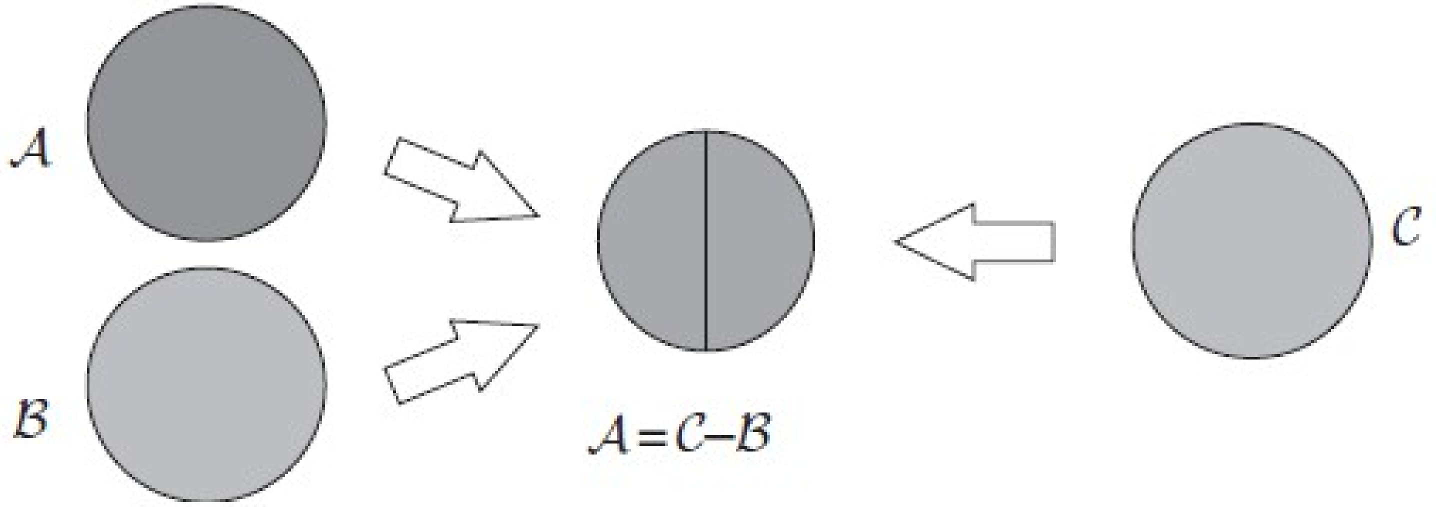


Equivalence classes

2 Beams of light that appear the same
(not necessary identical)

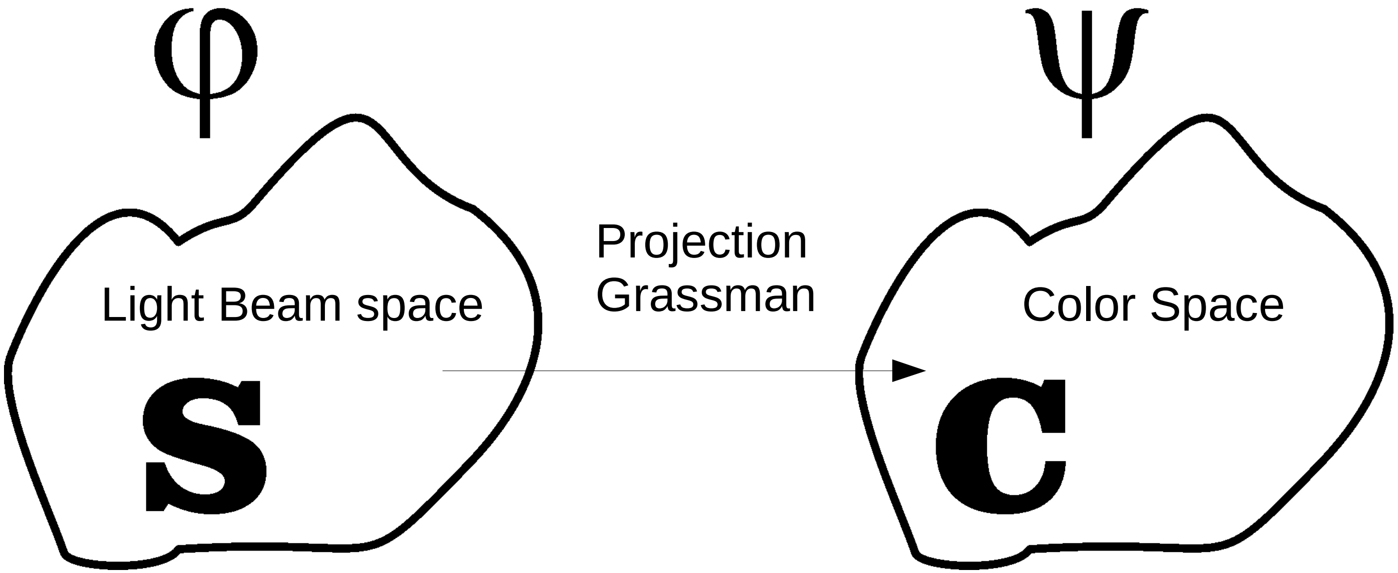
$$\mathcal{A} <=> \mathcal{B}$$

$$\mathcal{A} - \mathcal{B} <=> \mathcal{O}$$



$$\mathcal{A} + \mathcal{B} <=> \mathcal{C} \qquad \mathcal{A} <=> \mathcal{C} - \mathcal{B}$$

Colorimetry model



φ

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Light Beam space

S

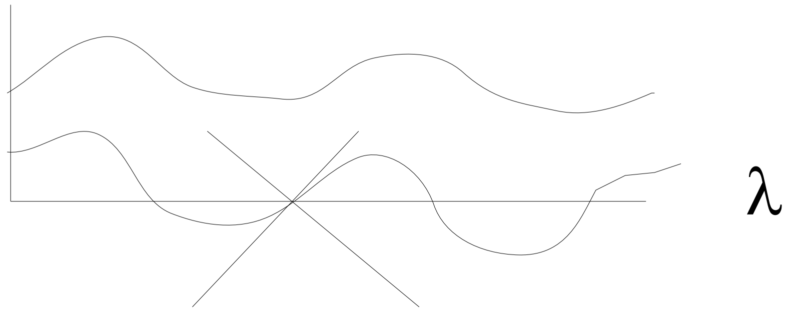
Projection
Grassman

Color Space

C

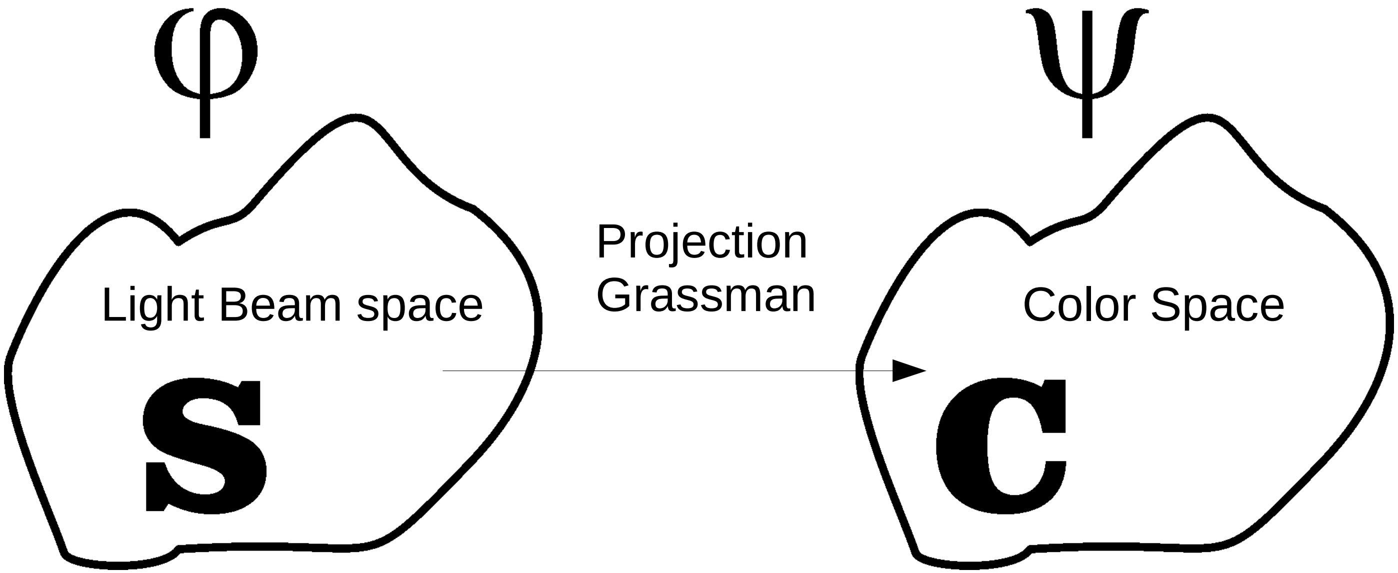
∞ dimensional space

3 dimensional space



Negative radiance not physically realisable

Colorimetry model



φ

ψ

Light Beam space

S

Projection
Grassman

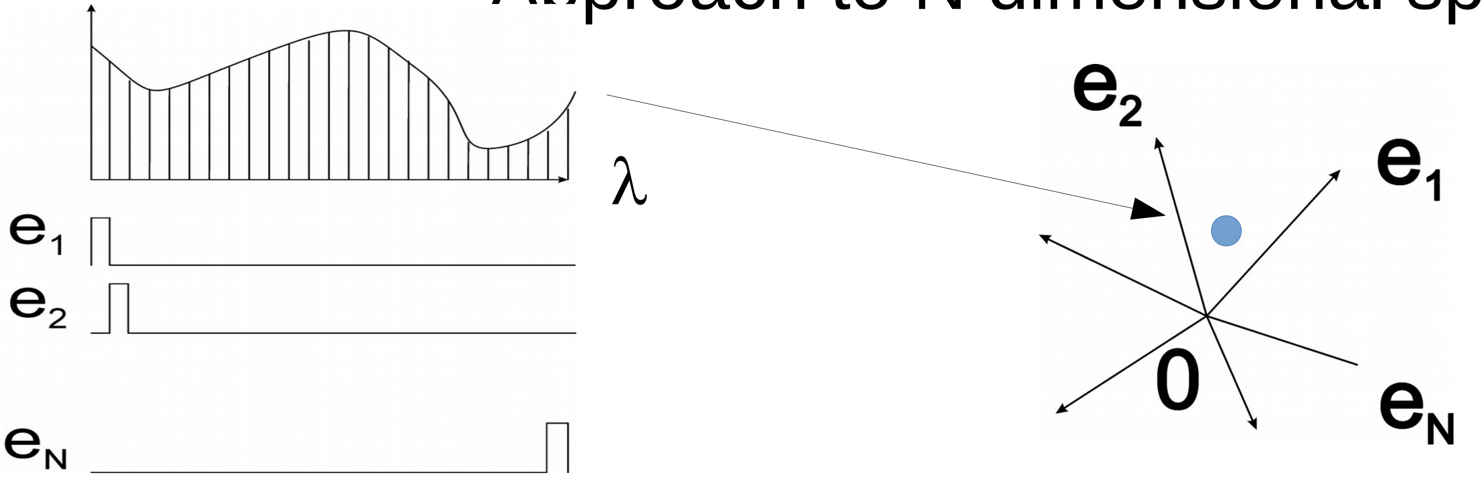
Color Space

C

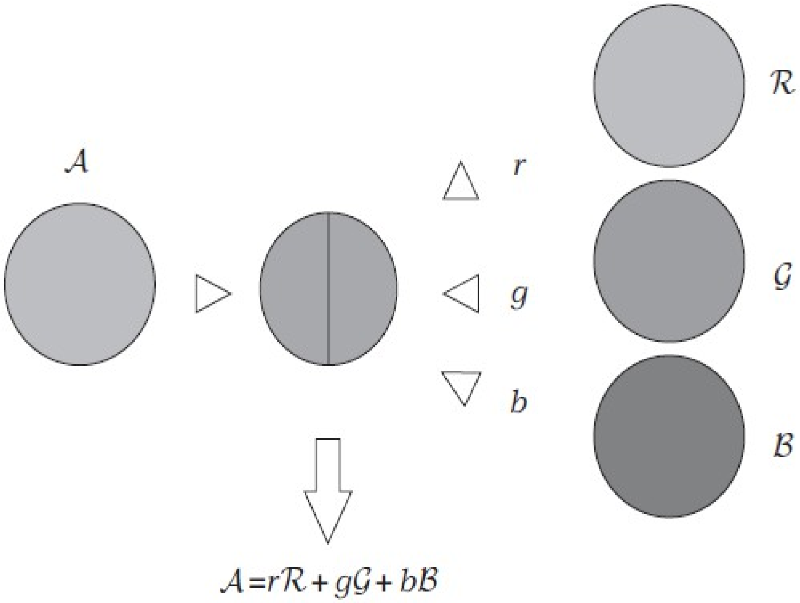
∞ dimensional space

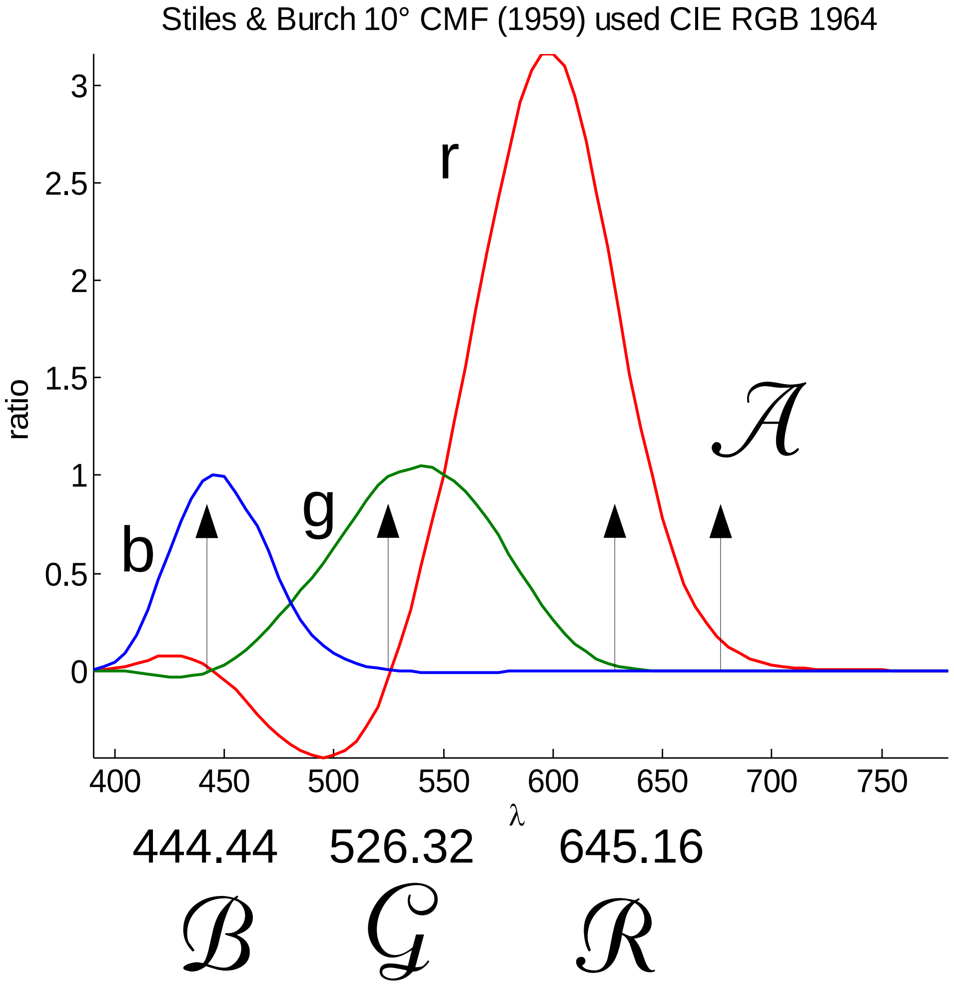
3 dimensional space

Approach to N dimensional space



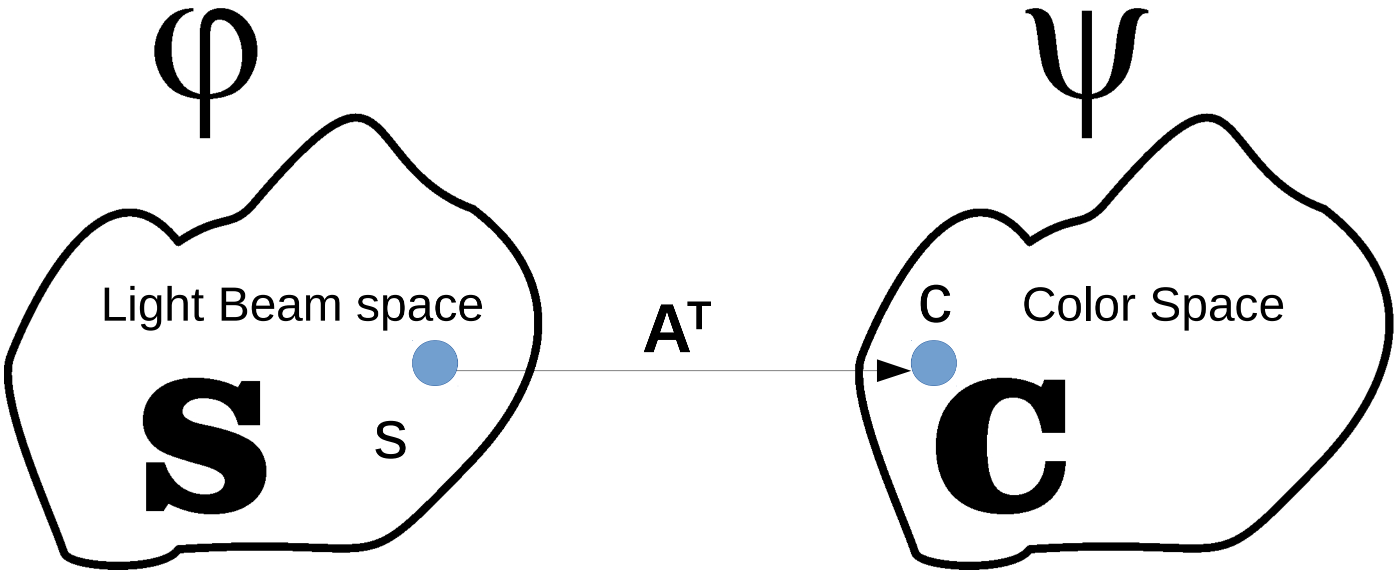
Gauging the spectrum





Matrix form $\mathbf{A} = [r(\lambda)\ g(\lambda)\ b(\lambda)]$

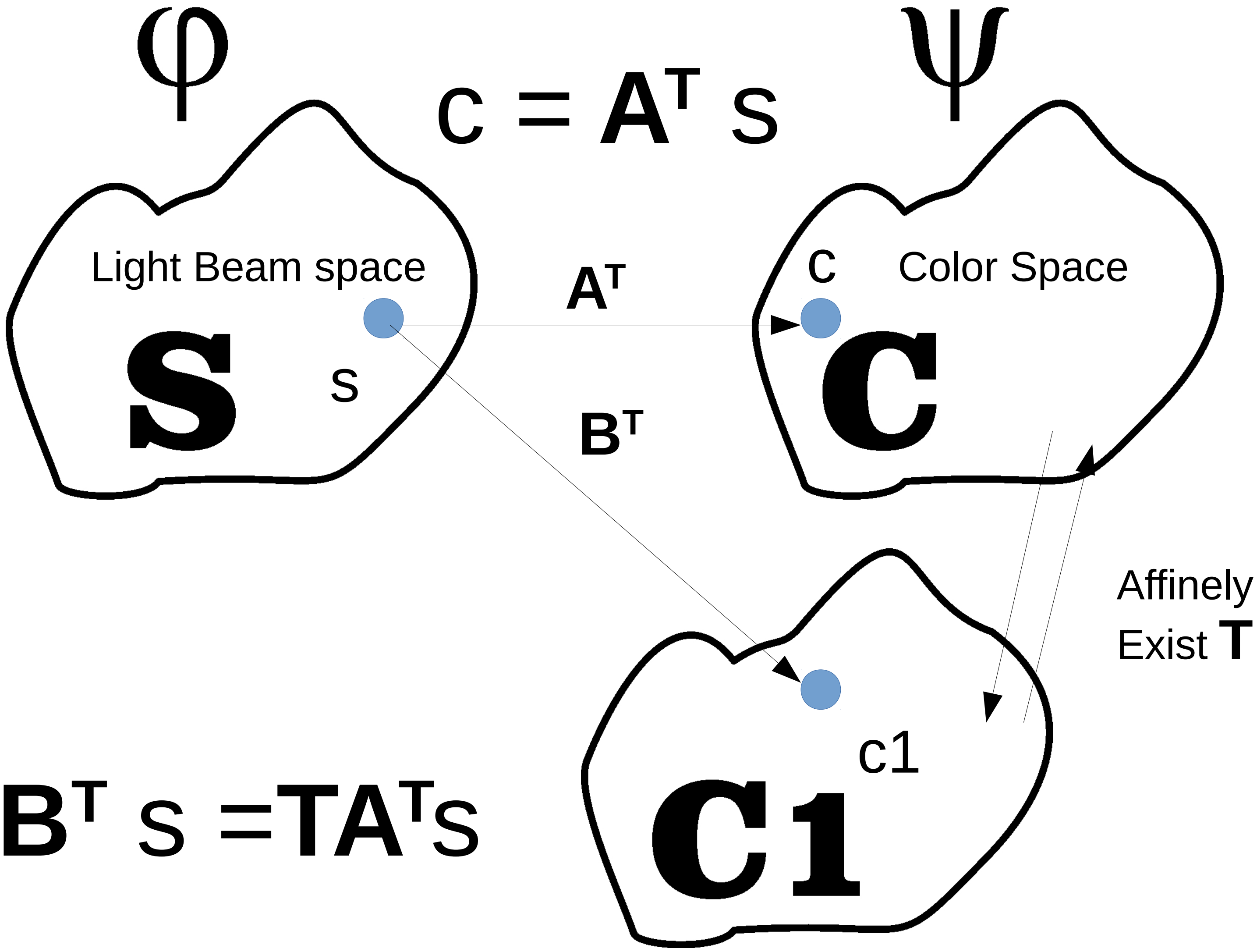
Colorimetry model



$S = (s_1,s_2,\dots s_N)^T$ $C = (c_1,c_2,c_3)^T$

$$c = \mathbf{A}^T s$$

Change in primaries



φ

$$c = \mathbf{A}^T s$$

ψ

Light Beam space

S

s

$\mathbf{A}^T$

$\mathbf{B}^T$

c

Color Space

C

c1

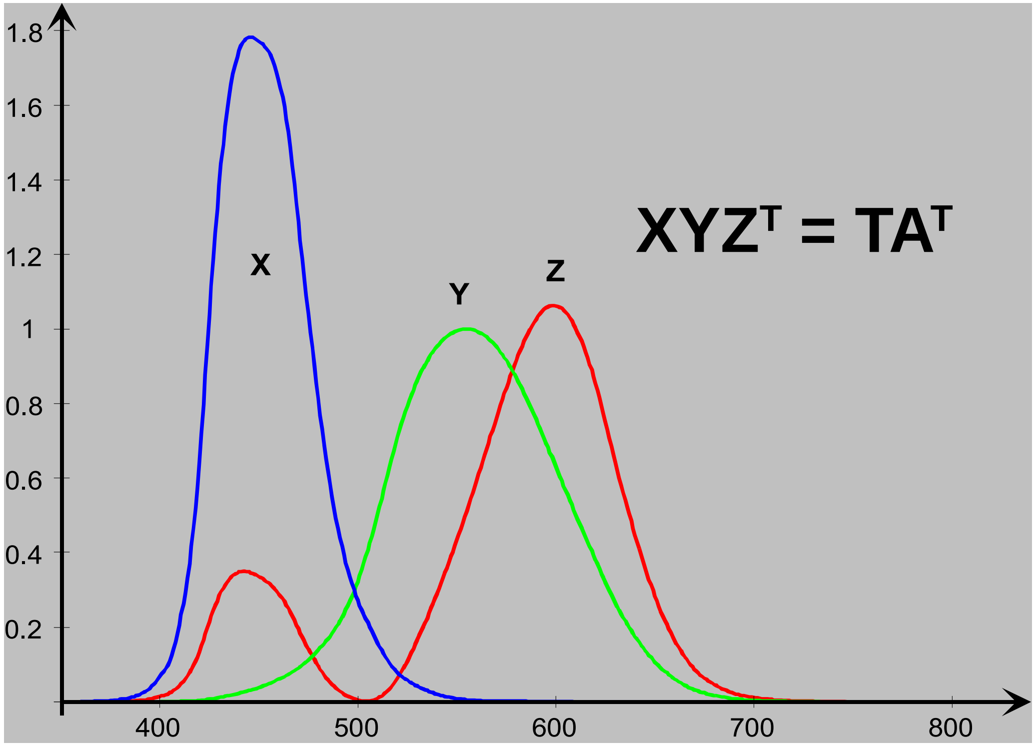
C1

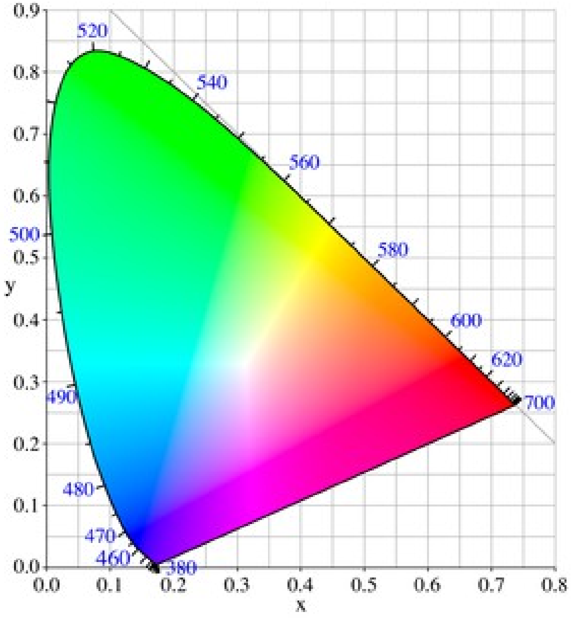
Affinely equivalent

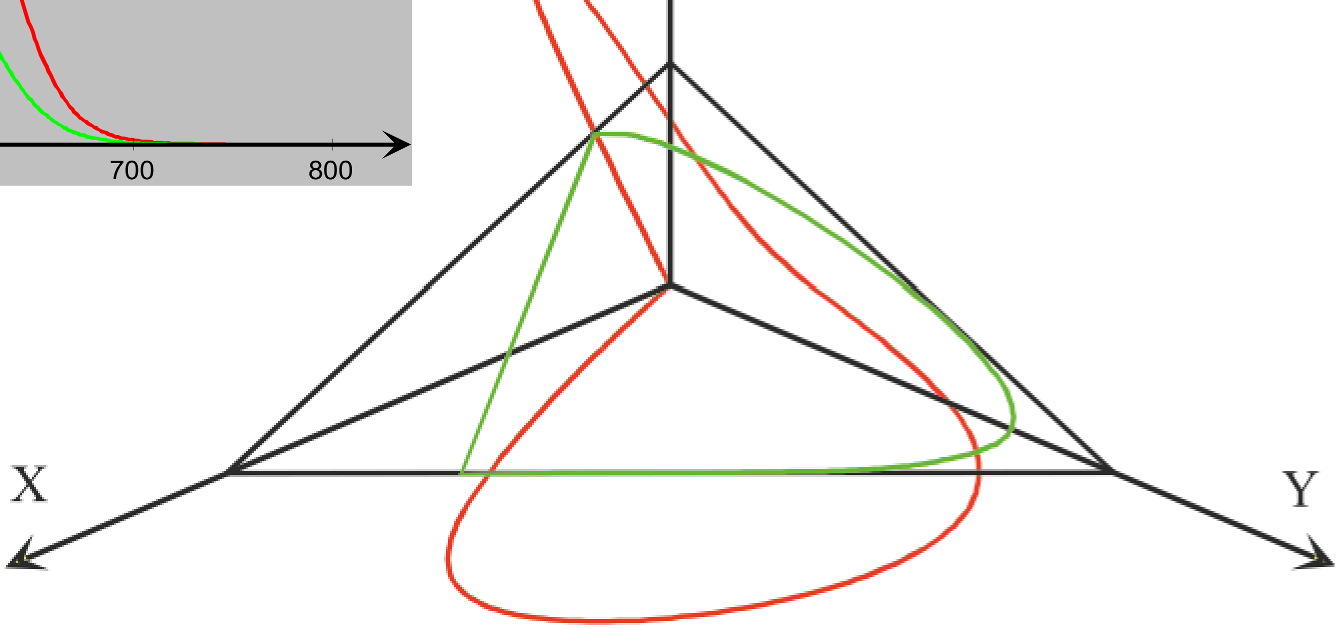
Exist **T** non singular

$$c1 = \mathbf{B}^T s = \mathbf{T}\mathbf{A}^T s$$

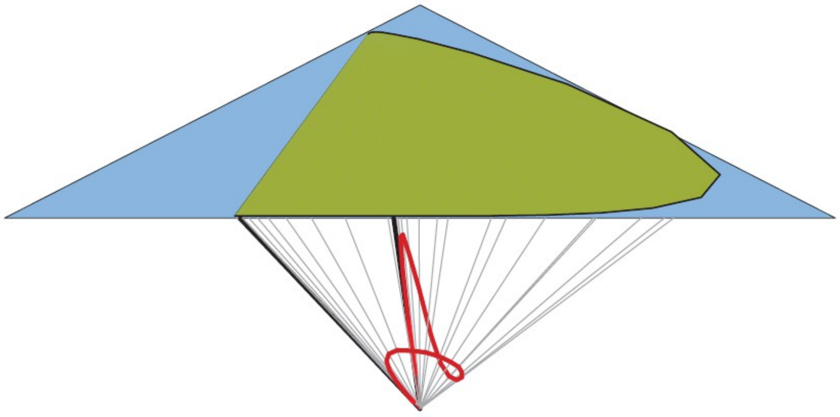
Spectral cone in CIE XYZ 1931

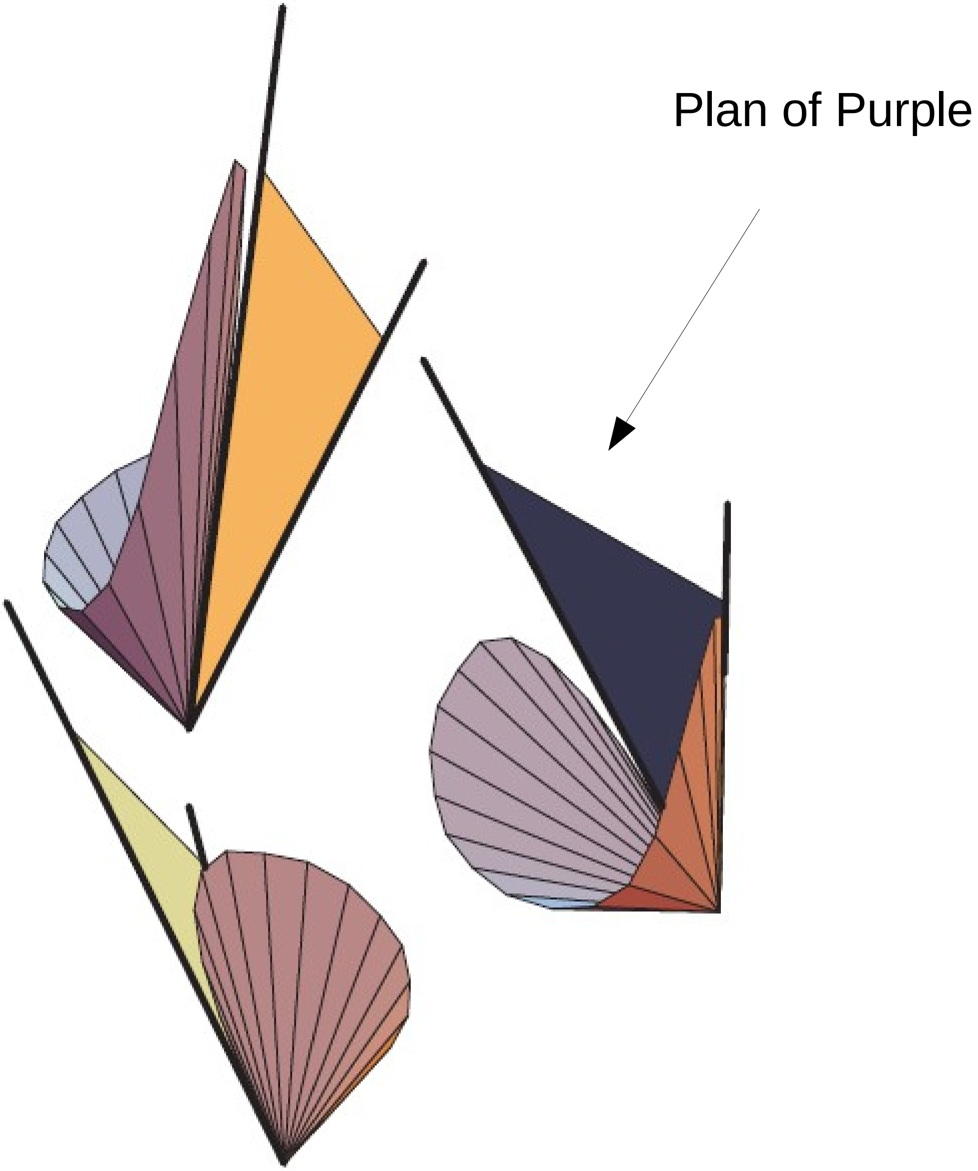






Spectral cone in CIE





Invariant in Color Matching Function

Wyszecki hypothesys :

Black

Radiometric function → $N = N^* + B$

Fundamental

N metamer N*

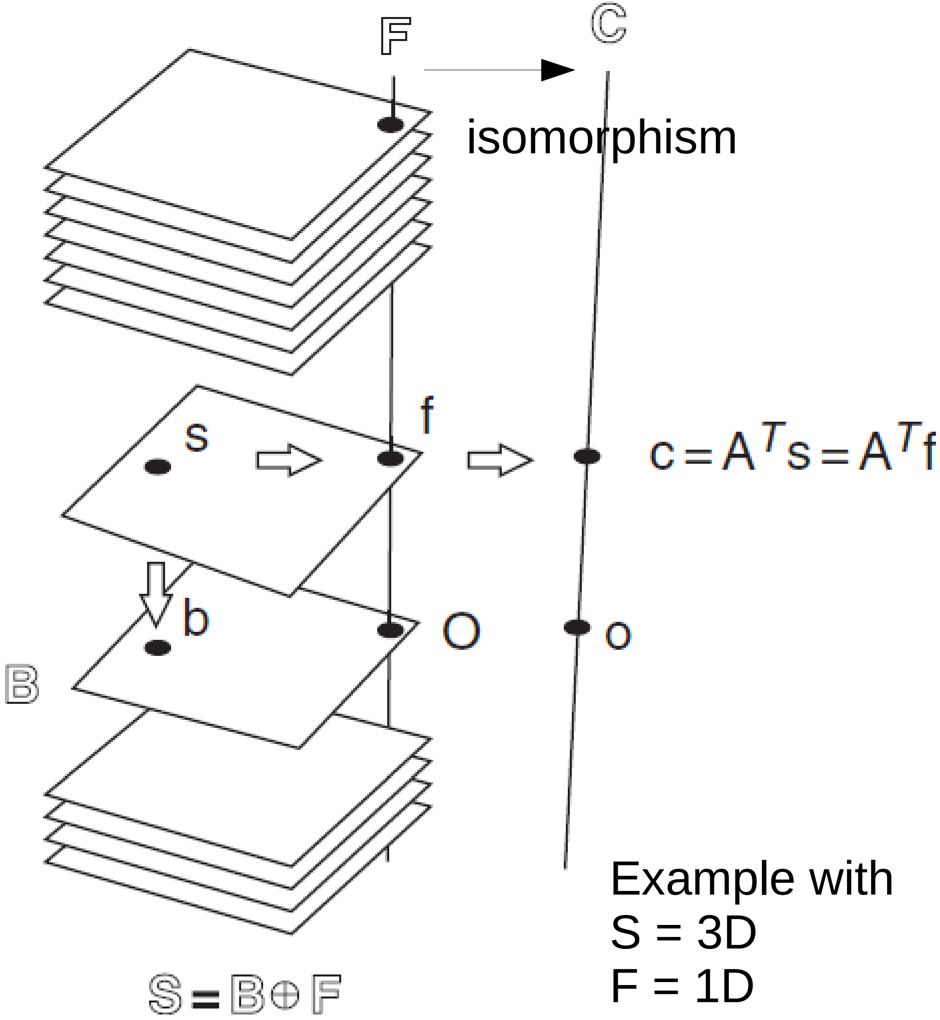
Space of beam ∞ dimensional space

S = F + B

Fundamental Space 3 dimensional

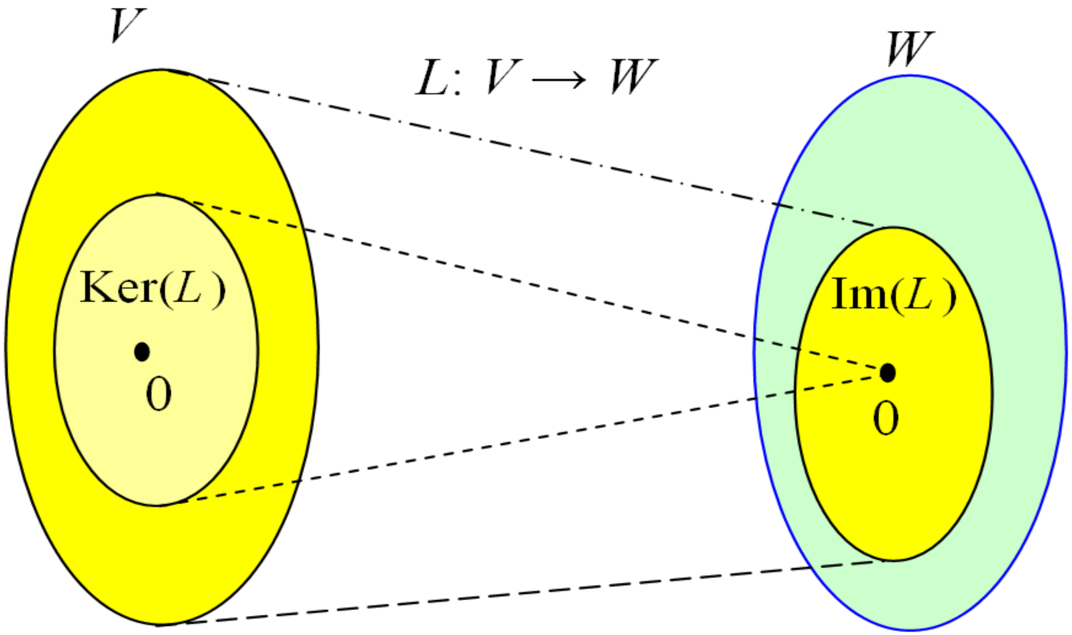
Black Space ∞ - 3 dimensional

Fiber Bundle



Example with
S = 3D
F = 1D
B = 2D

Linear Algebra



$v_1 <=> v_2$

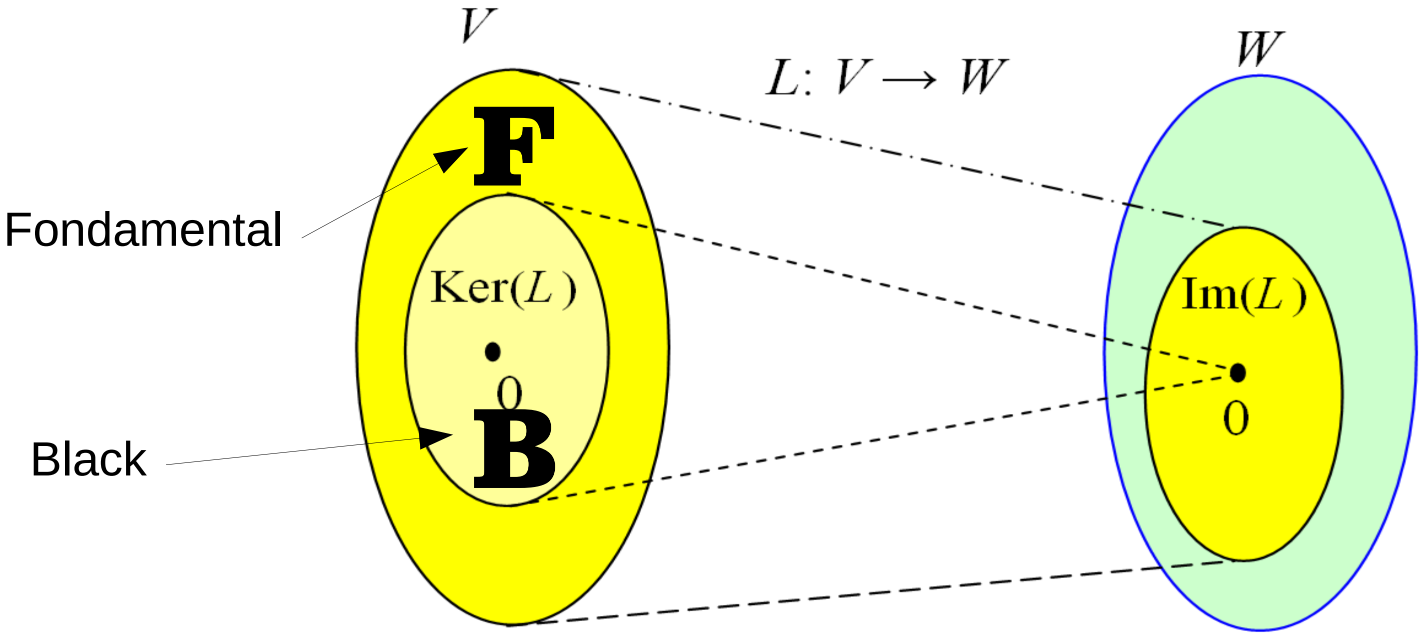
$L(v_1)=L(v_2)$

$L(v_1-v_2)=\mathbf{0}$

im(L)= V/Ker(L)
espace quotient

dim(Ker(L)) + dim(im(L)) = dim(V)

Linear Algebra



S = V

$L(\mathcal{V}_1)=L(\mathcal{V}_2)$

$\mathcal{V}_1 <=> \mathcal{V}_2$

$L = \mathbf{A}^T$

W = C

$L(\mathcal{V}_1-\mathcal{V}_2)=\mathbf{0}$

dim(Ker(A)) = dim(S)-dim(Im(A)) =∞ - 3

Cohen R matrix

R project a beam to the fundamental space, R_b to the black

$\mathbf{P} = [P_1\ P_2\ P_3]$ Primaries such that A^TP_1, A^TP_2, A^TP_3

Linearly independent

$\mathbf{A}$ CMF of human under P

$$\mathbf{A^t}s = c = \mathbf{A^t}f \qquad <=> \qquad \exists\ \mathbf{R_B} / \mathbf{A^tR_B}=0$$

$$\mathbf{A^tP = 1 = P^tA}$$

$$\mathbf{(A^tA)(A^tA)^{-1} = 1 = A^tP}$$

$$\mathbf{A(A^tA)^{-1} = P}$$

$$\mathbf{P(P^tP)^{-1} = A}$$

$$\mathbf{R=PA^t=AP^t}$$

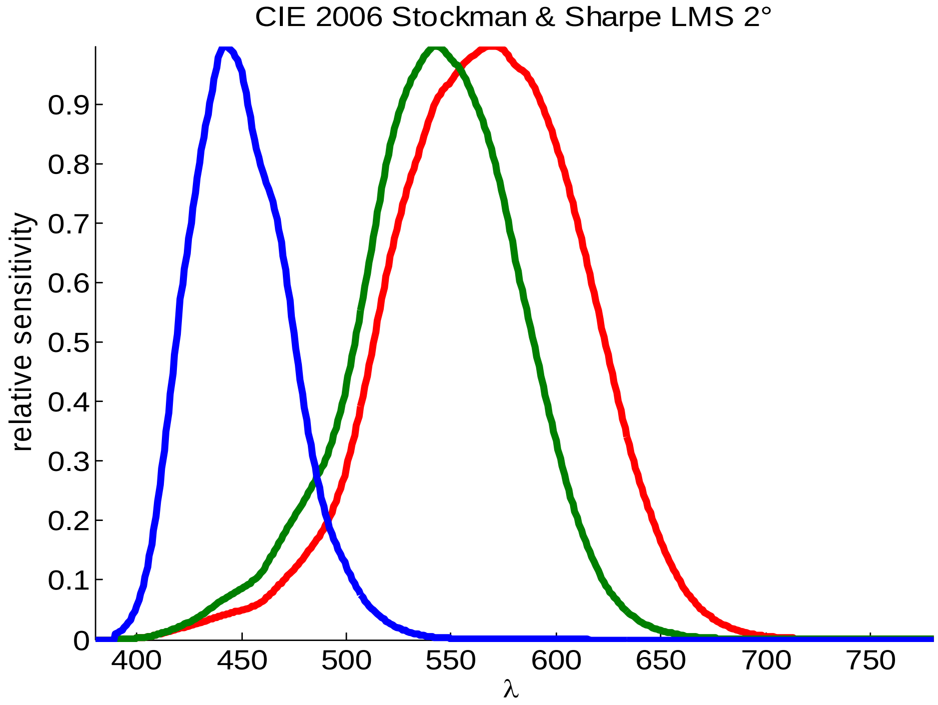
$$\mathbf{R_B=I-R}$$

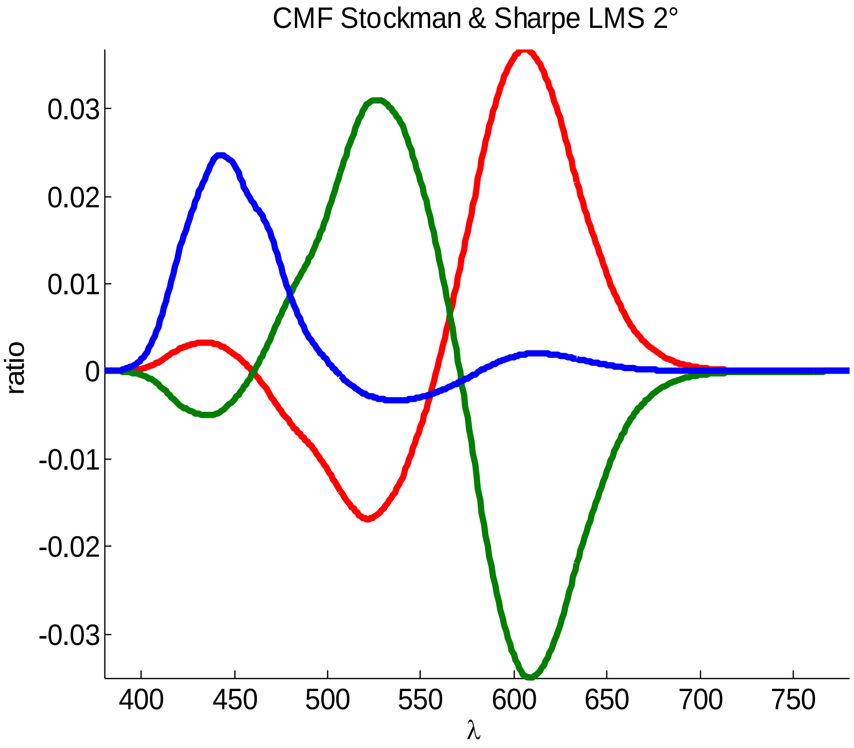
Human Colour Vision

Cone's excitation LMS as primaries

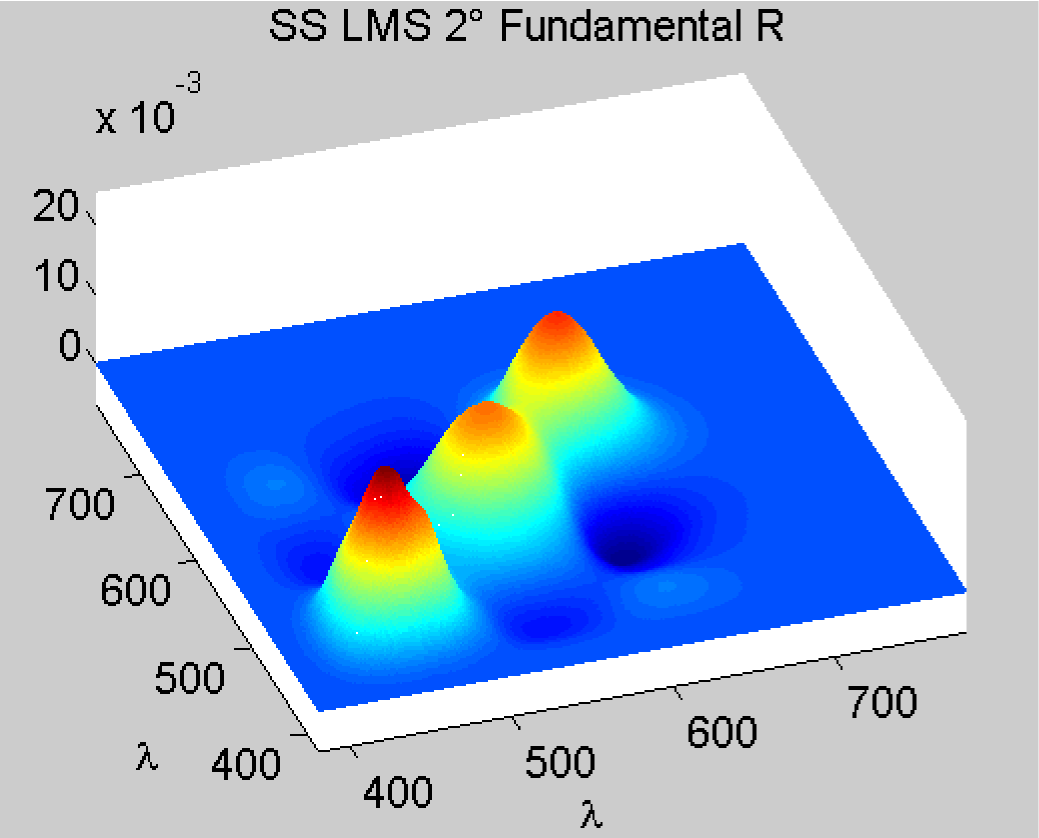
$P = [L\ M\ S]$ Nx3

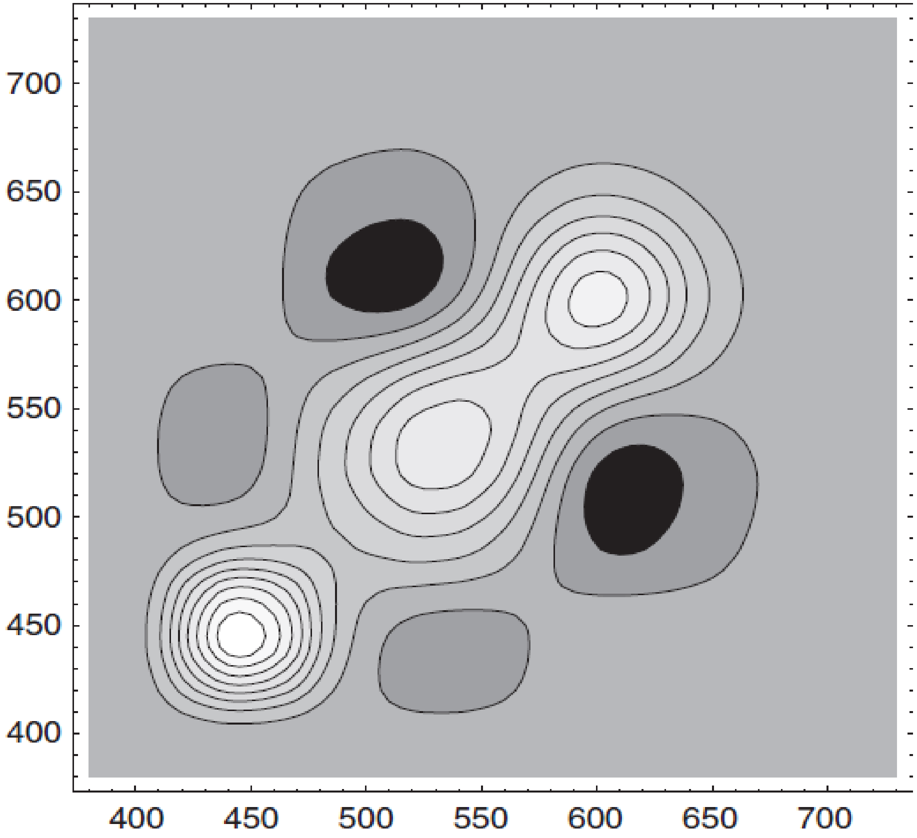
$A = P(P^tP)^{-1}$





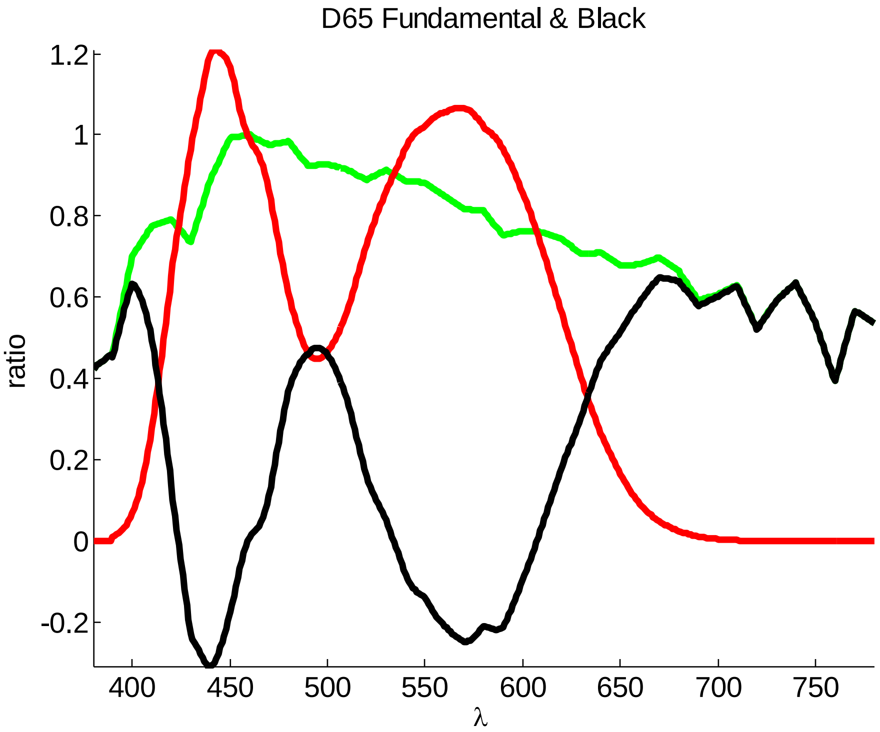
Corresponding R


SS LMS 2° Fundamental R

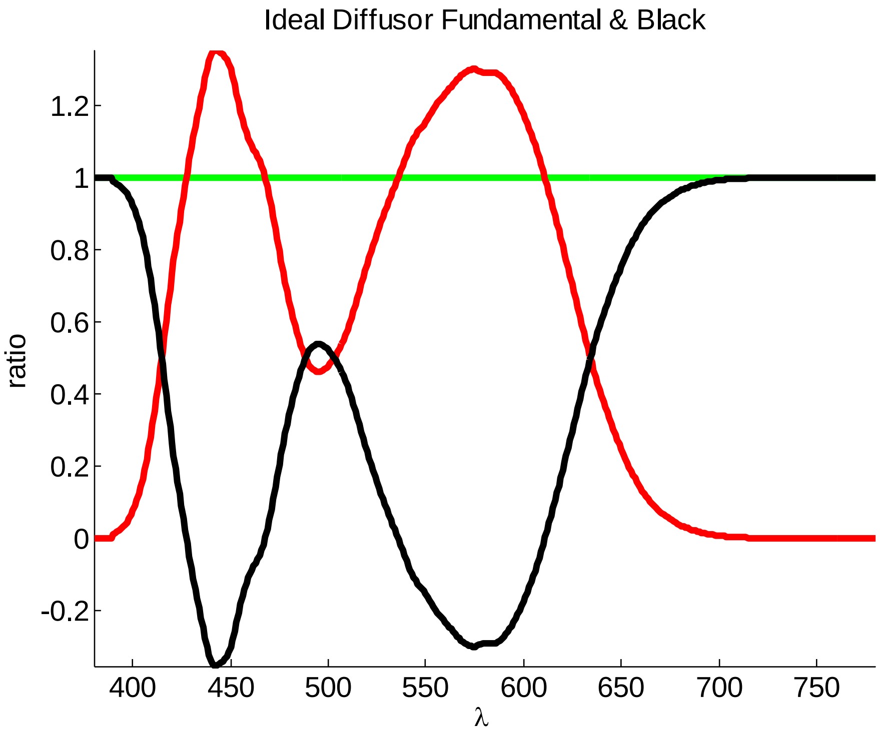


Example of Fundamental calculus

D65F=R'*D65 D65B=RB'*D65



Ideal Diffusor

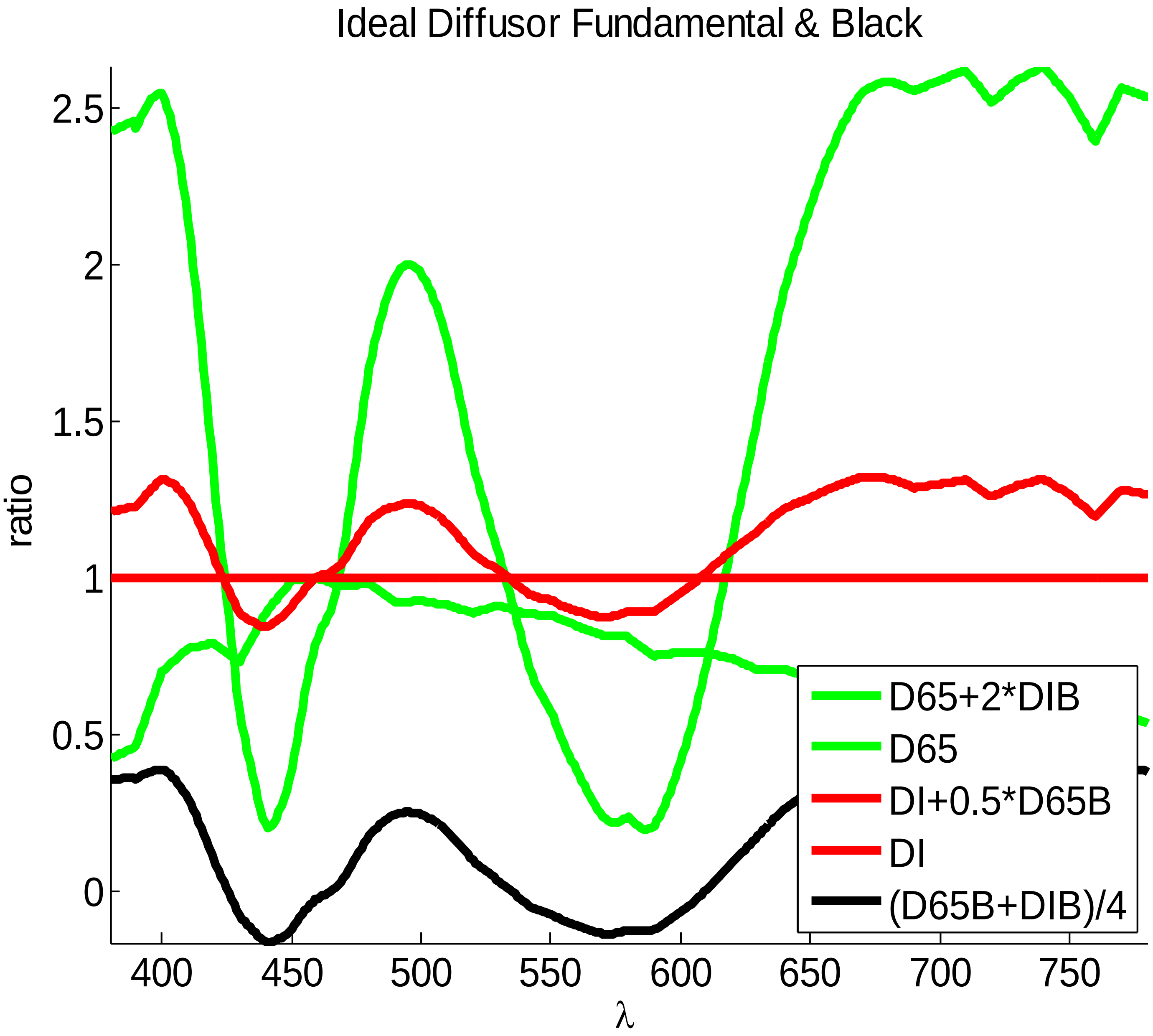


P'*D65= 95.9503
82.1712
52.7008

P'*D65F= 95.9503
82.1712
52.7008

P'*D65B= 1.0e-012 *0.1534
0.1411
0.0984

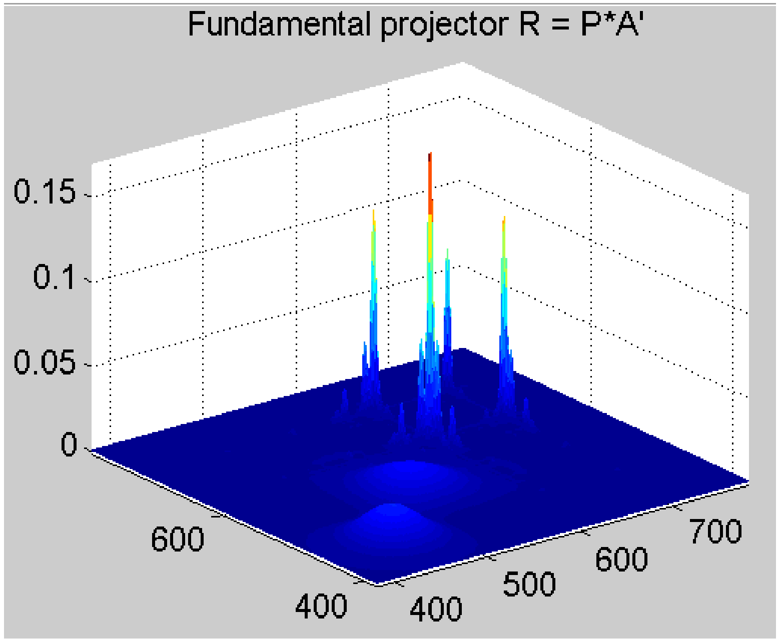
Modelling black space

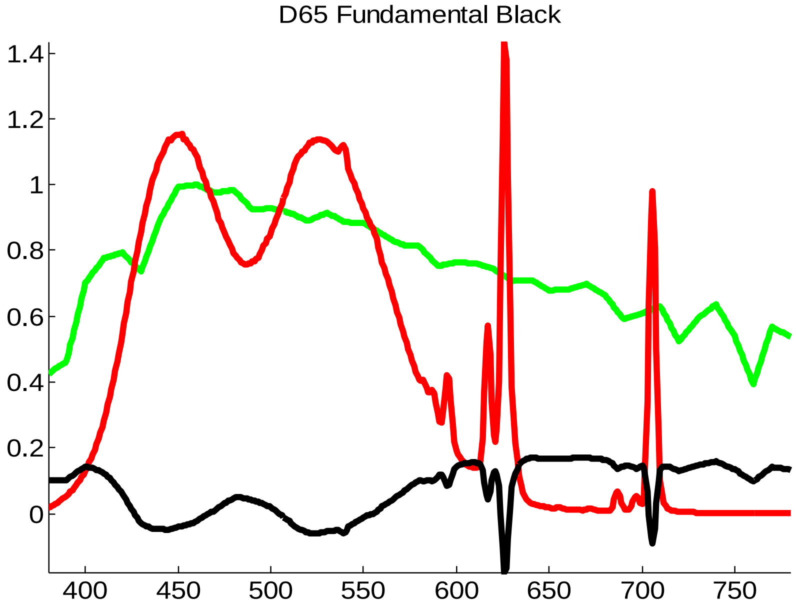


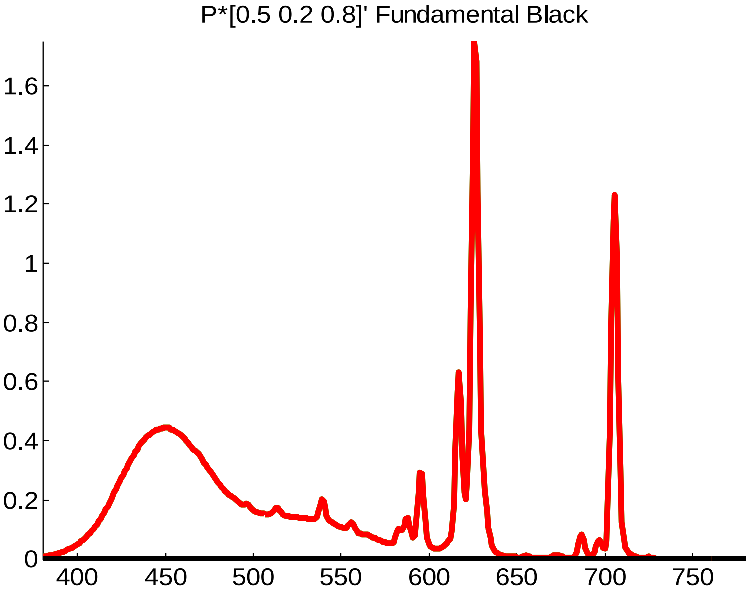
P'*(D65+k*DIB)=P'*D65

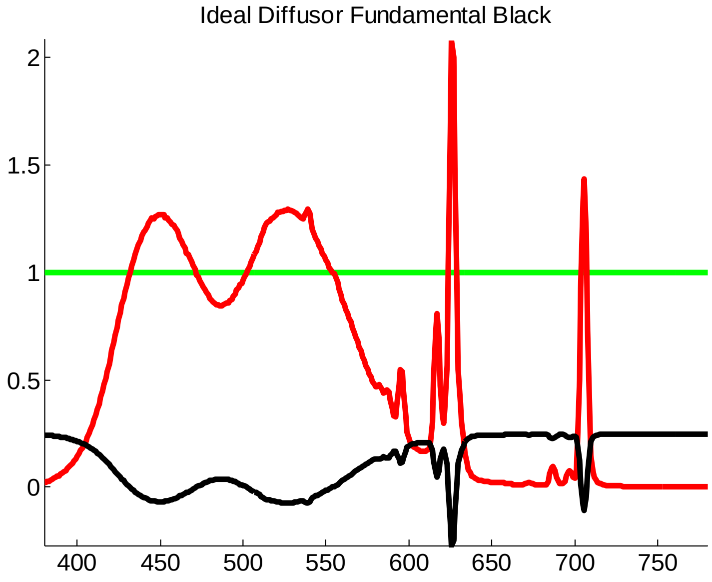
D65+k*DIB <=> D65

Example Screen


Fundamental projector R = P*A'

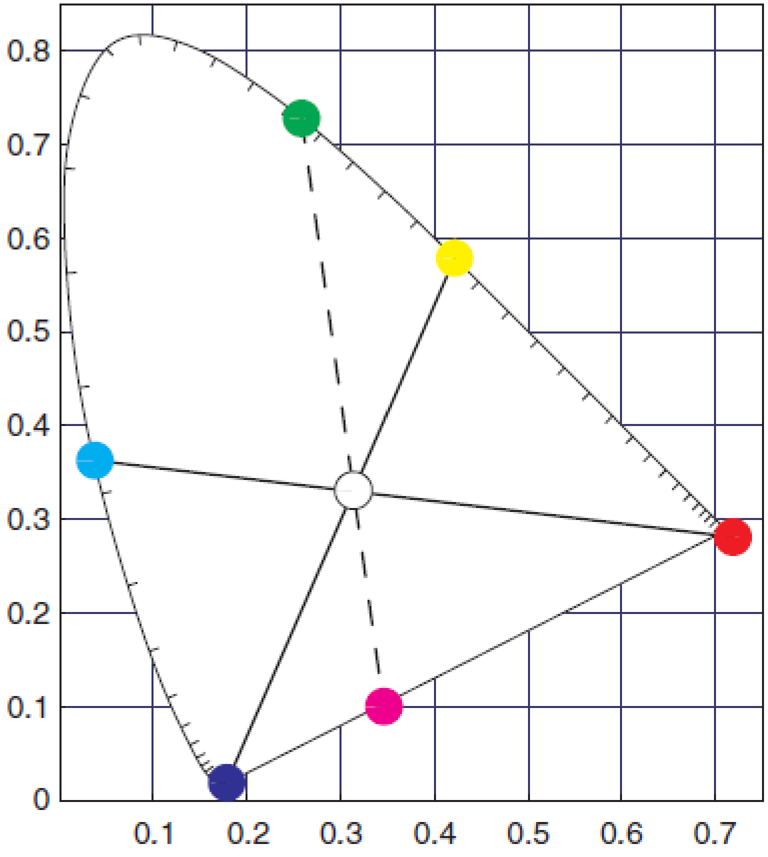

D65 Fundamental Black


P*[0.5 0.2 0.8]' Fundamental Black

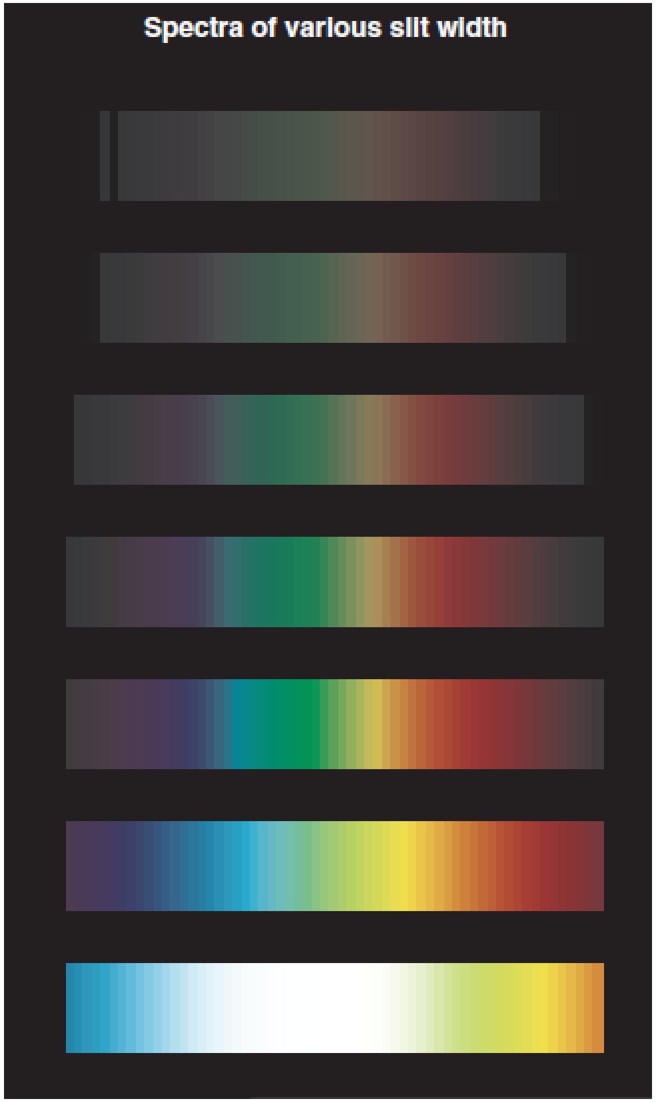

Ideal Diffusor Fundamental Black

Koenderink application

Cold / Warm color

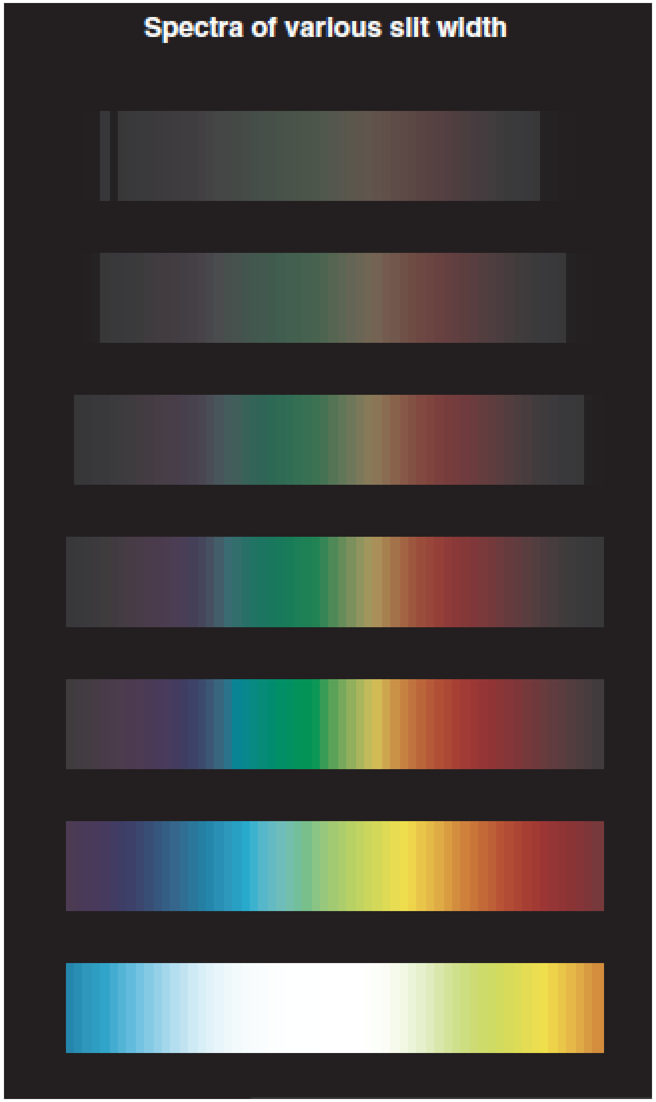


Cold

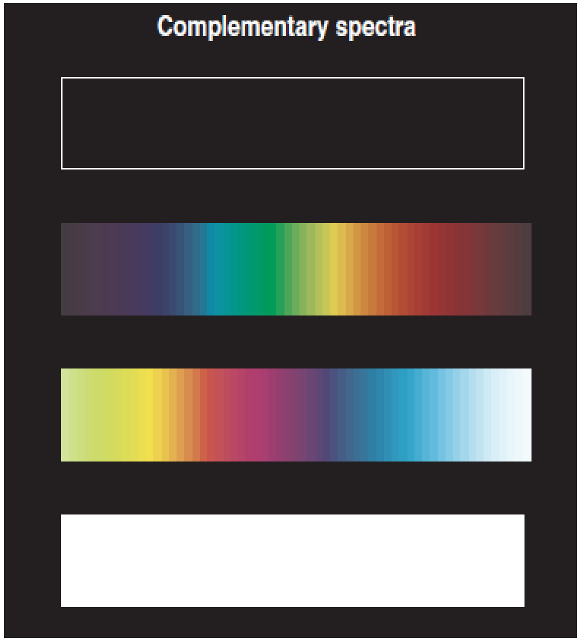


Warm

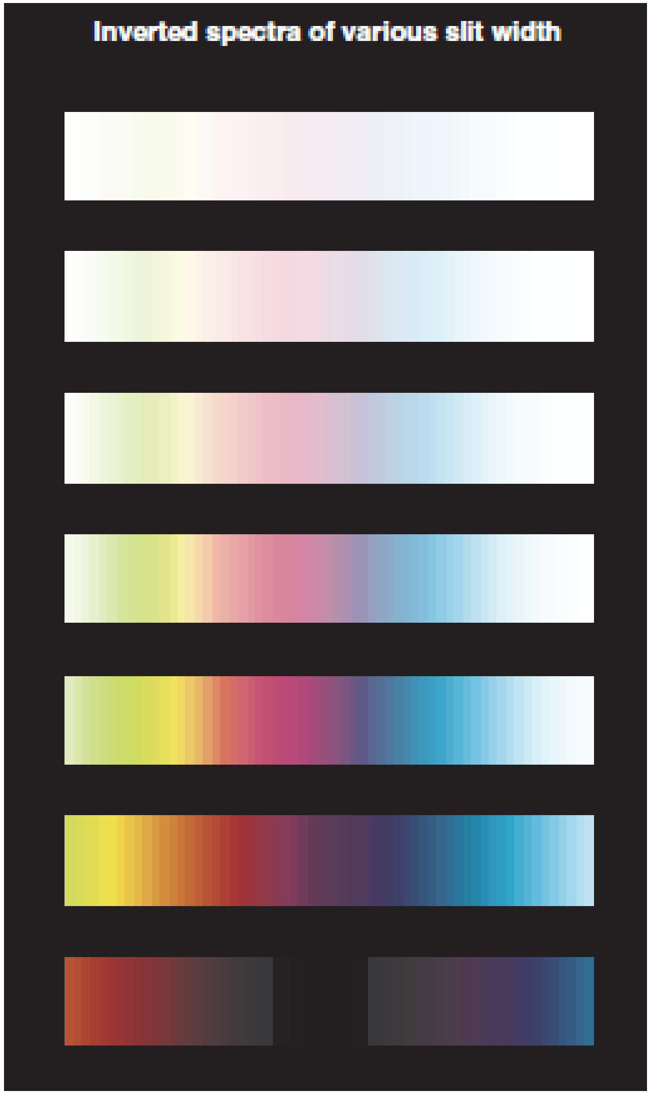
Koenderink application



Spectra of various slit width

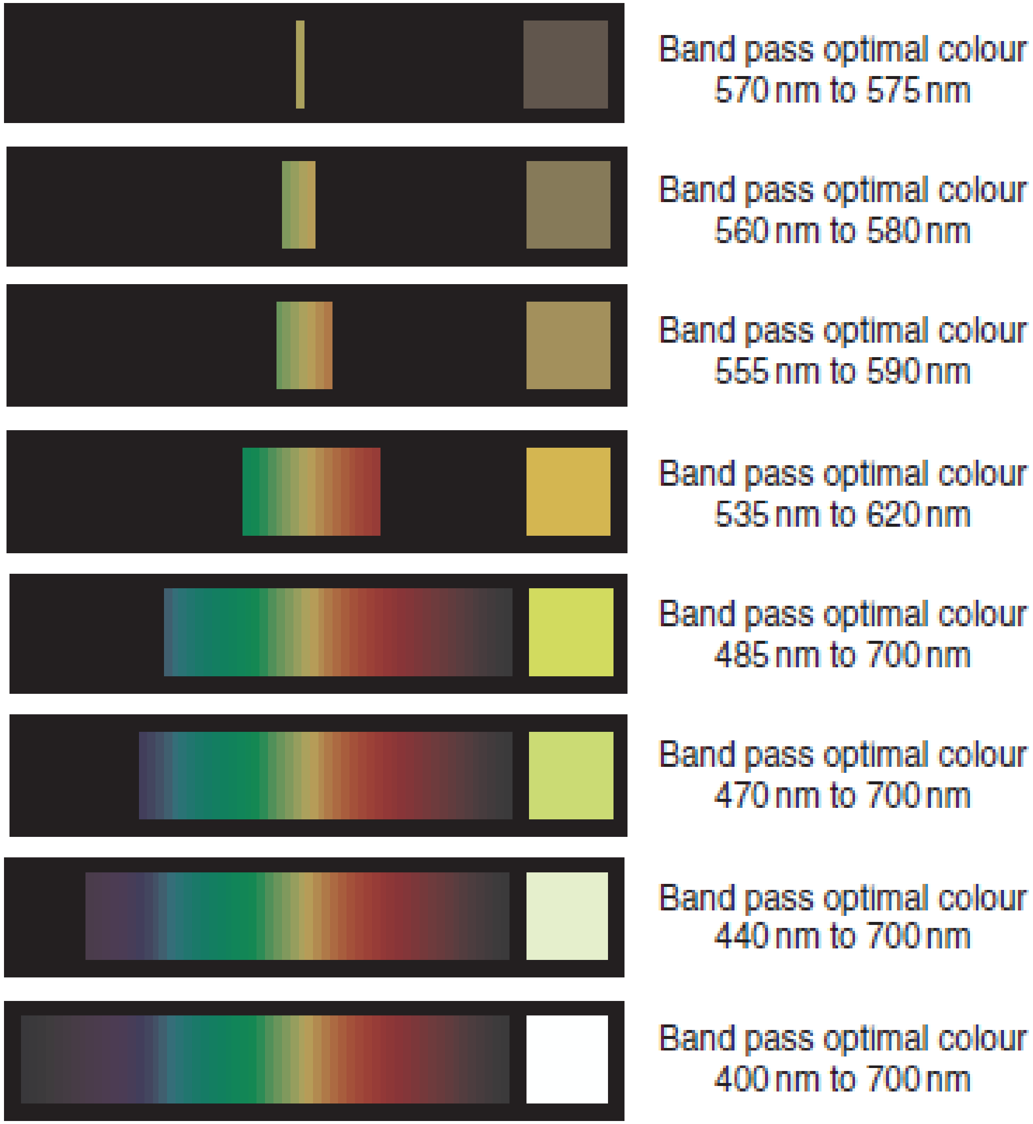


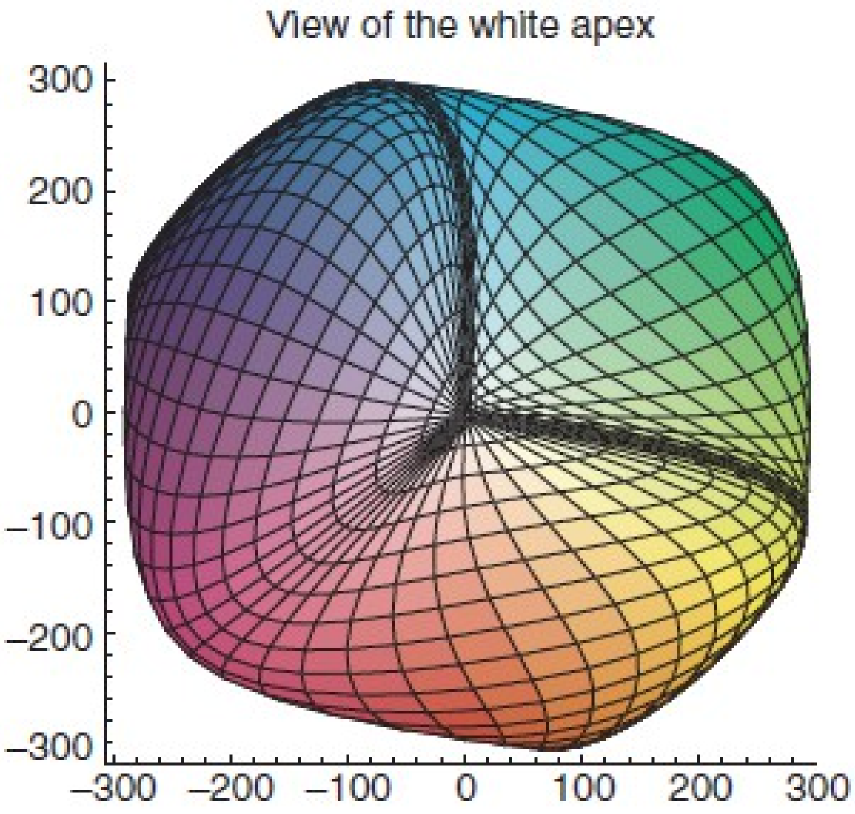
Complementary spectra



Inverted spectra of various slit width

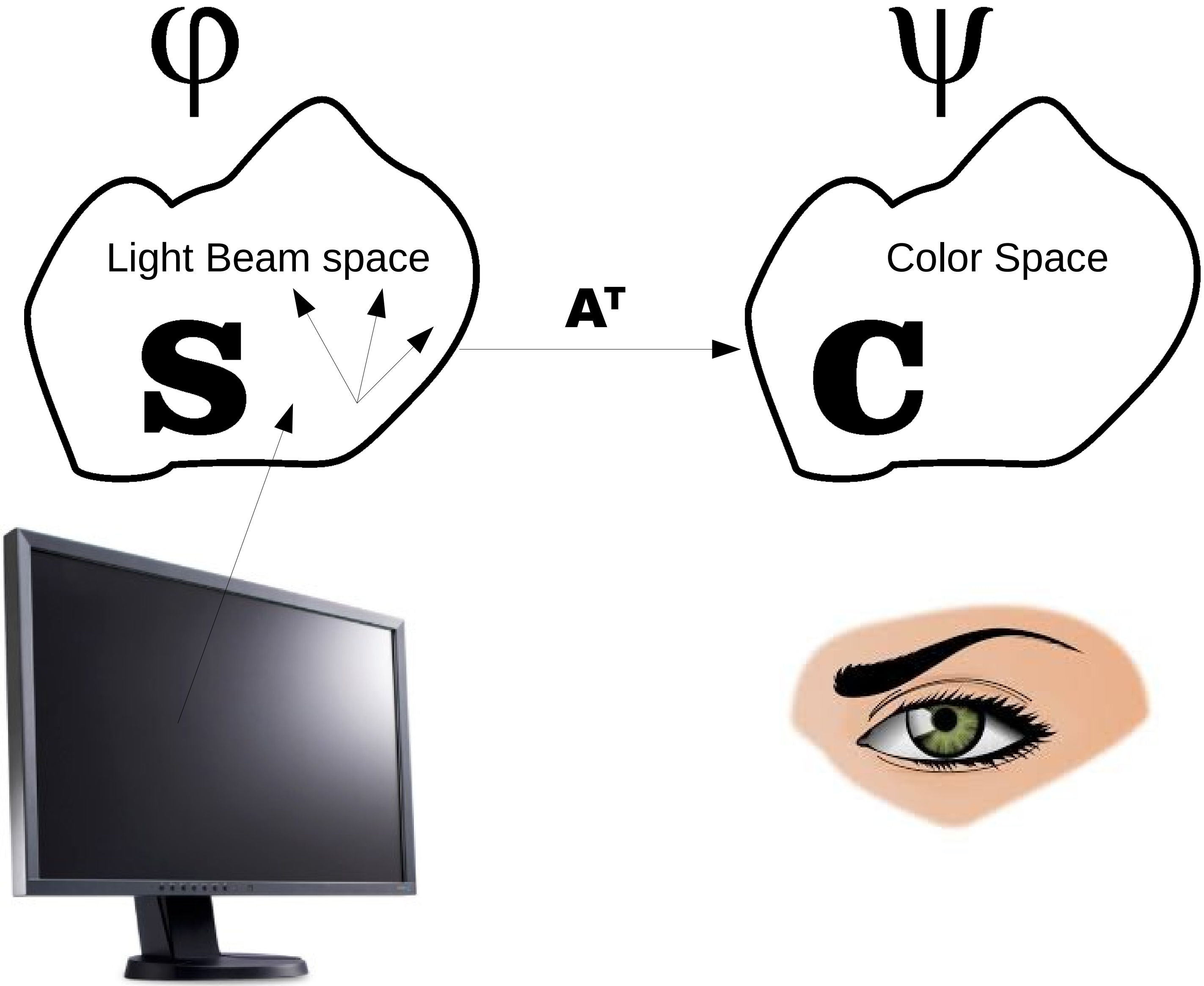
Koenderink application



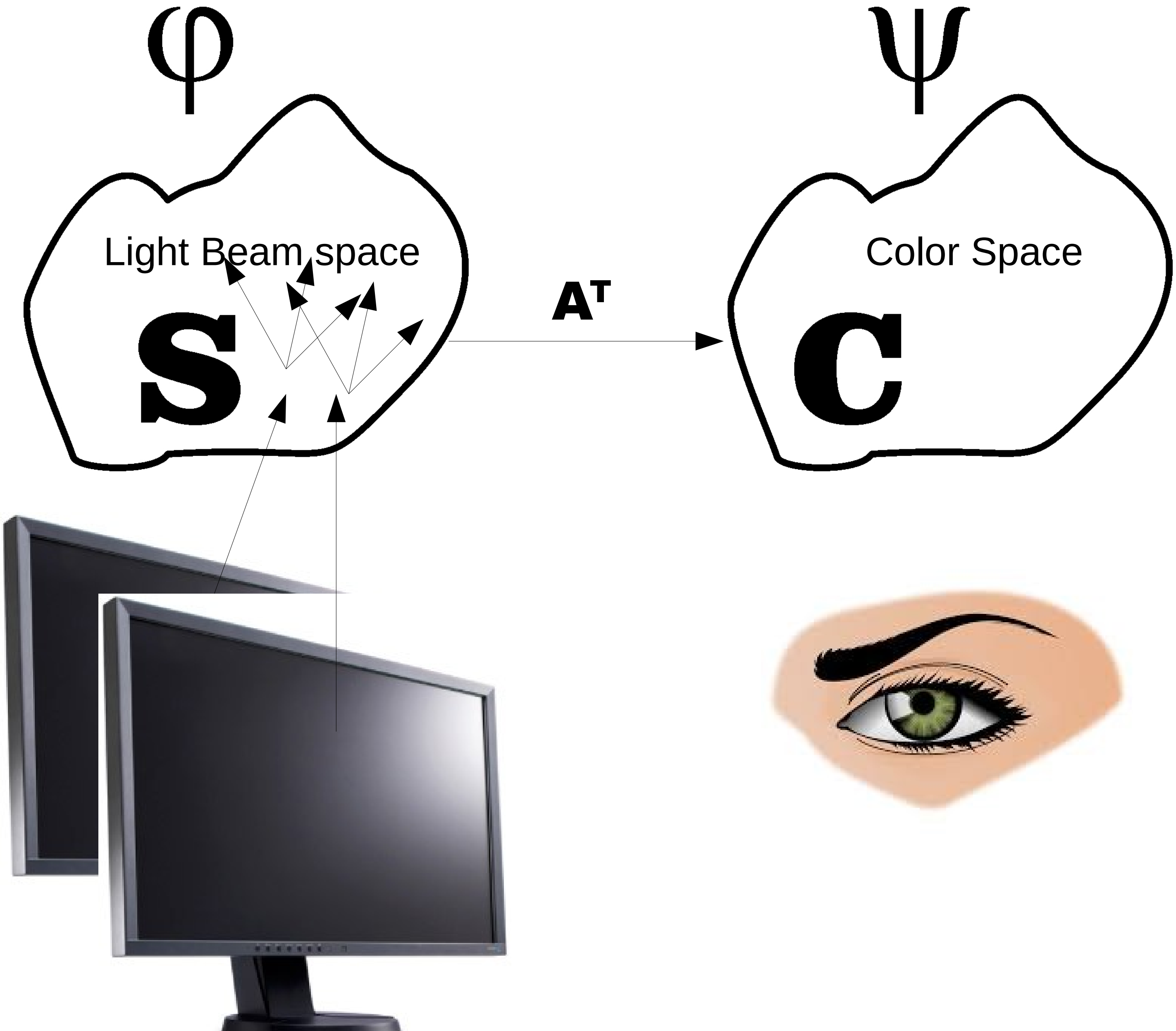


Schrödinger

Cognitive experiment



2 different screens



φ

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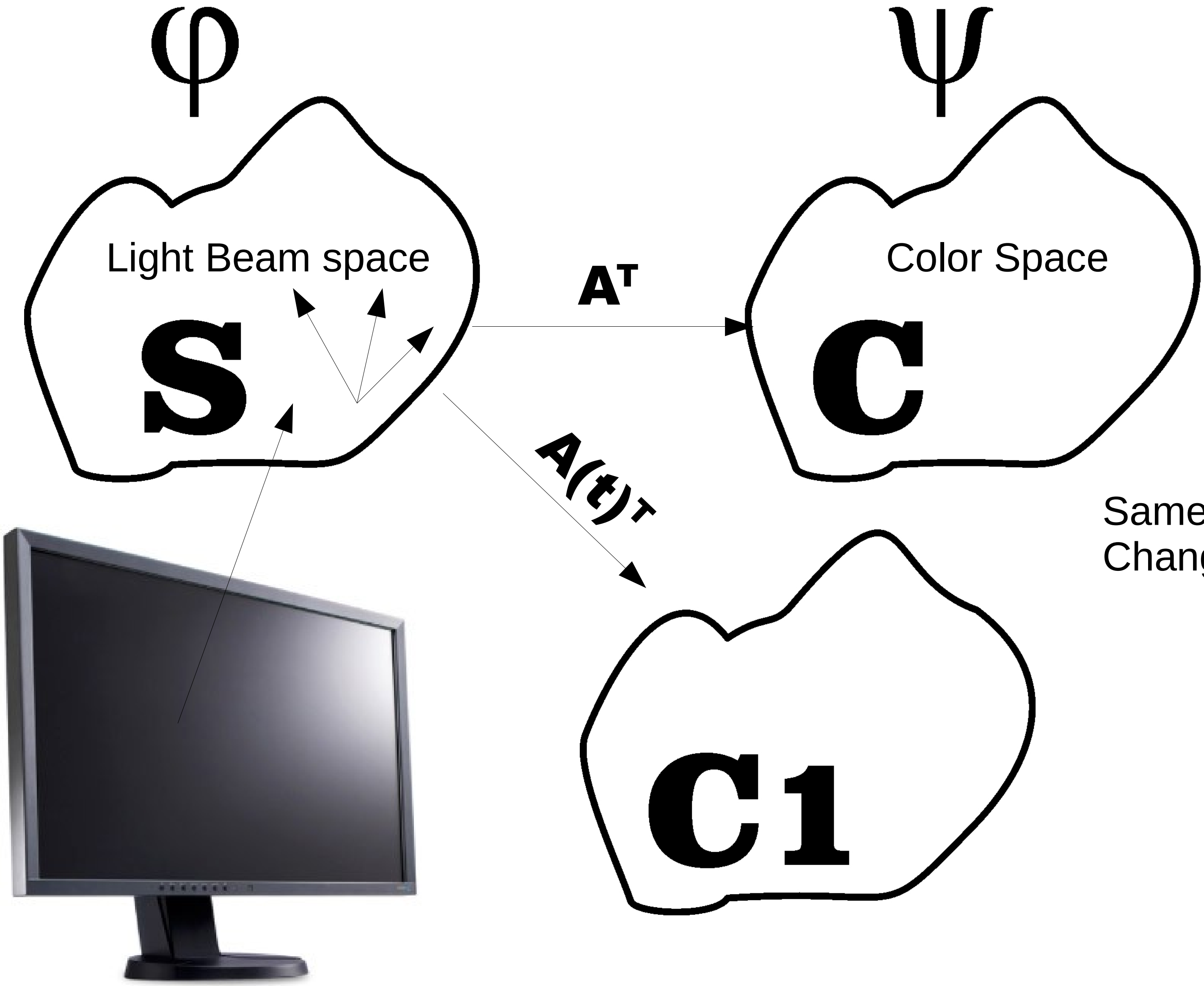
Light Beam space

Color Space

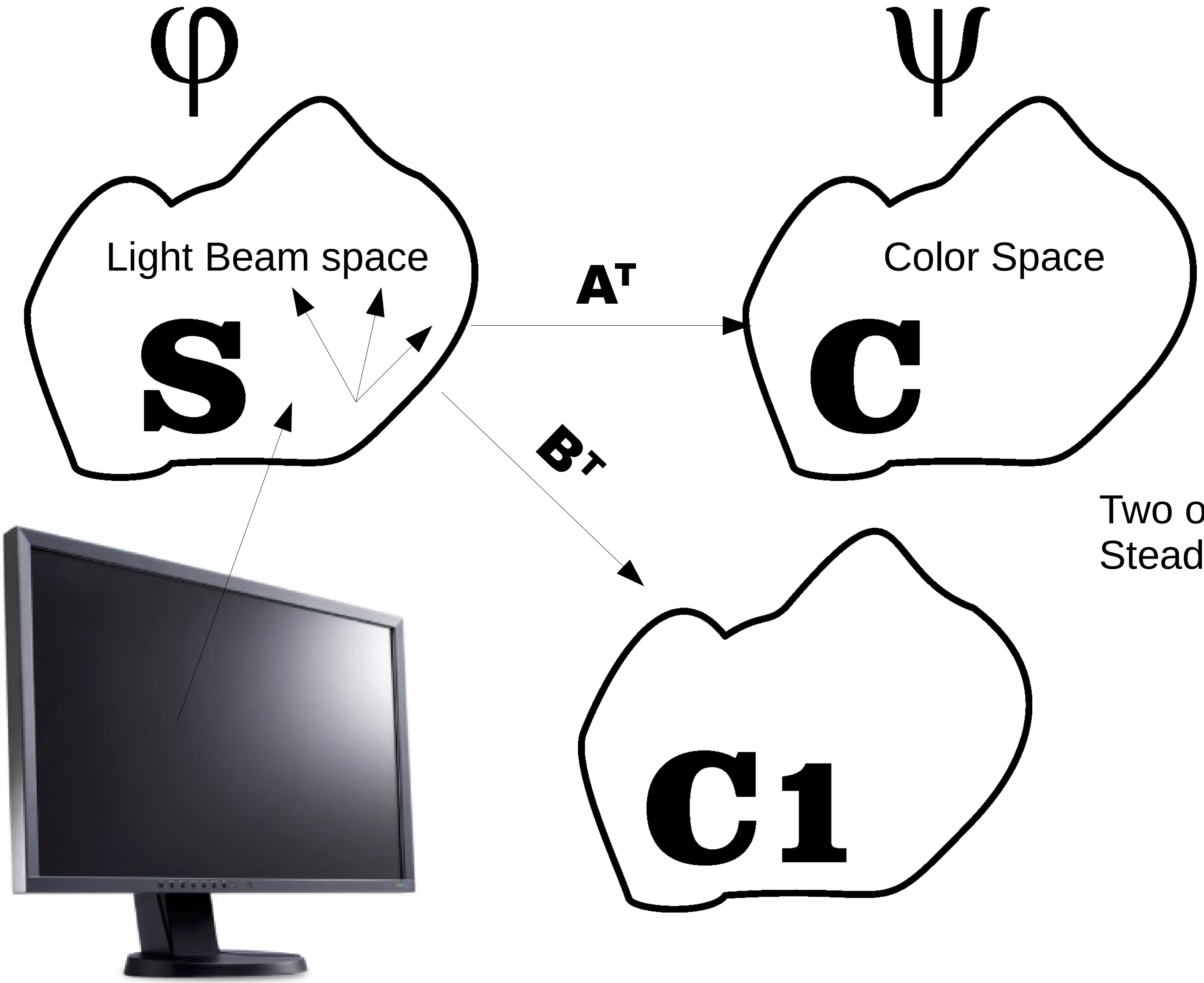
$\mathbf{S} \xrightarrow{A^T} \mathbf{C}$

Adaptation



Same observer
Changing state

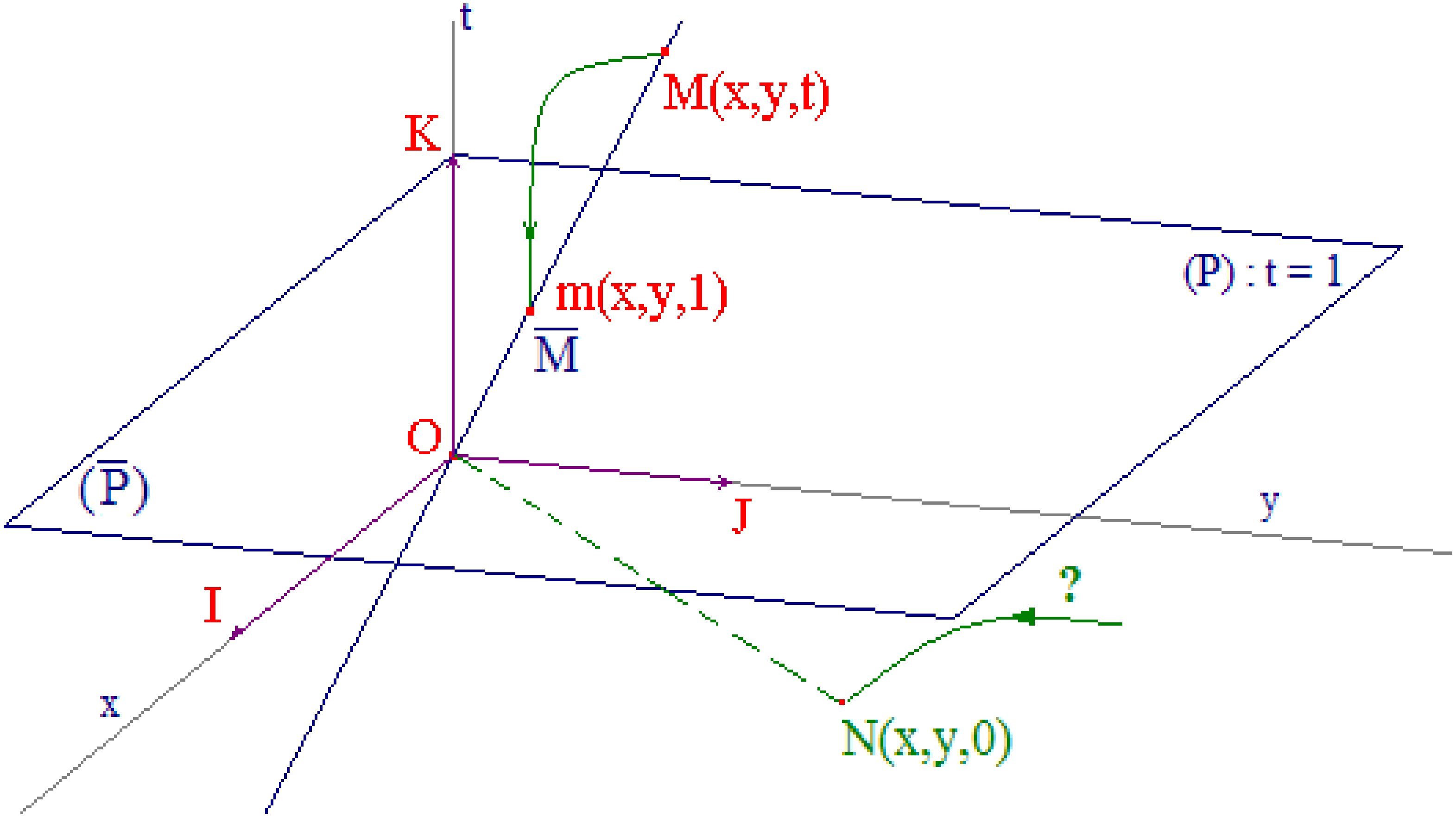
Inter-individual differences



Two observers in
Steady state

Conclusion

- Color and linear algebra for everybody
- Check the steady state as availability to produce same threshold for observers on two different screens
- Find equivalence classes for other sense
- Control for metric apparatus/observer


t
M(x,y,t)
K
(P) : t = 1
m(x,y,1)
$\overline{M}$
O
$(\overline{P})$
J
y
?
I
x
N(x,y,0)